tripleC i(i): pp-pp, year
ISSN 1726-670X
http://tripleC.uti.at

Inference Rules, Emergent Wholes and Supervenient Properties

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Abstract: Computer images are "emergent wholes" in relation to their pixels. This may seem to suggest that there cannot be any valid formal inference rule connecting such images with their constituents. However, there is one; one that applies to many kinds of emergent wholes. The rule is extracted from R.M. Hare's writings on supervenience, and is here baptized *the supervenience rule for emergent wholes*. This rule is distinct from both *the plain rule for emergent wholes*, which is invalid, and David Lewis' corresponding rule and concept of supervenience, which do not differentiate between cases of reduction and non-reduction. In philosophical ontology, Lewis' definition of "supervenience" obliterates the distinction between emergent and non-emergent entities, while in informatics and computer science it may complicate computing.

Keywords: image retrieval, inference rule, emergent whole, supervenient property, David Lewis

Acknowledgement: This paper was written under the auspices of the Wolfgang Paul Program of the Alexander von Humboldt Foundation, the Network of Excellence in Semantic Interoperability and Data Mining in Biomedicine of the European Union, and the project Forms of Life sponsored by the Volkswagen Foundation.

1 Introduction

In many corners of science and philosophy, the term "emergent whole" has for some reason got a very negative ring, as if it would refer to some occult mumbo-jumbo quality. But this is wrong. In the processes of evolution, completely new physical and chemical substances, new geological formations, new biological organisms, and new qualities have entered the world. That is, evolution has given rise to "emergent wholes" and "emergent properties". Such emergent entities are normally able to exist independently of what they have emerged *from*. This is not the kind of emergence that will be discussed in this paper. Here, the term "emergent whole" will be reserved for entities that in a sense emerge *on* something, their *base*, on which they are always dependent for their existence and with which they thus always have to exist simultaneously. The perceptual Gestalten studied by Gestalt psychology – often said to be "more than the sum of their parts" – are the classic examples of such emergent wholes (Smith 1988). In relation to emergent wholes there are, I will show, two *formal inference rules* that ought to be of interest in informatics and computer science.

Formal rules can either be explicitly presented as rules or be implicitly presented by means of hypothetical statements that are assumed to be necessarily true. For instance, the classical rule of *modus ponens* can be presented either as the rule

- From (p and (if p then q)) conclude q;
  or as the logical necessity
- Necessarily, if (p and (if p then q)), then q.

The necessity statement states the logical truth that makes the inference rule formally valid; it may be said to contain the rule in question implicitly. Mostly, I will use such necessity statements in order to present the inference rules that will be discussed.

2 Emergent Wholes and Concrete Collections

What is characteristic of emergent wholes can best be captured in a comparison with a kind of entity that I will call "concrete collections". Look at what is inside the brackets [- ˙ ˙)]. Here is a collection with four members: one black straight-line mark, two black dot-shaped marks, and one black right-parenthesis-shaped mark. However, there is more within the brackets than a collection of marks; there are also specific spatial relations between these marks. The collection of the four marks together with all their mutual spatial relations (as, for instance, represented by distances and places in a local coordinate system) is an example of a *concrete collection*. In a mere (or abstract) collection, all spatial relations are abstracted away. If the concrete collection [- ˙ ˙)] is replaced by [ ) ˙ - ˙], then there is a new concrete collection but the same collection. A concrete collection consists of some spatiotemporal individuals (its members) and their monadic qualities plus the mutual spatial relations between these individuals.

Concrete collections have both a numerical and a qualitative identity. One and the same concrete collection can endure, i.e., it can retain its (numerical) identity in time. It might even be able to move and be situated in different places at different times; this requires that both its members and their mutual spatial relations stay the same. If two numerically different concrete collections have the same number and kind of members in the same kind of spatial relations, then they are qualitatively identical, i.e., they are concrete collections of the same kind.

A concrete collection is more than a mere collection, but it is not an emergent whole. As a first example of an emergent whole, I will use the well-known face of the basic computer smiley :-). Of the two concrete collections (a) [- ˙ ˙)], and (b) [:-)], only (b) is a base for an emergent whole. In (b), there are not, as in (a), only four black shapes and their mutual spatial relations, there is also a smiling face. A smiley is more than the sum of four individual marks and some spatial relations; it is something that *emerges on* such concrete collections. An emergent whole coincides in space – more or less – with the concrete collection that is its base. (The same is true in relation to time, but, for the sake of brevity of this paper, everything that has to do with time is disregarded.)

Every smiley has parts that belong to one of (at least) two simultaneously existing ontological levels, a base level and a supervenient level. On the one hand there are the parts of the concrete collection, and on the other there is the supervening smile that cannot be found in the concrete collection as such. Even though a smiley cannot possibly exist without some concrete collection, the concrete collections which can be its base need not be of exactly the same kind as in (b) above. Both the monadic properties of the collection members and their spatial relations may be somewhat different. In this sense, both the qualitative and the numerical identity of a smiley have some relative independence in relation to the concrete collection at hand.

The distinction between a base level and a supervenient level takes away the aura of mysticism that surrounds the phrase "being more than the sum of its parts". How can something possibly be more than the sum of its parts? In my opinion, it cannot – when the expression "its parts" is taken to mean "all its parts", and "part" is taken to mean not only spatial and temporal parts but also monadic qualities of the entity in question. An emergent whole is more than the sum of its parts on the base level, and it is more than its supervenient part, but it is identical with – and constituted by – the sum or fusion of the all parts of all levels.

Our usual distinction between a word and its meaning relies implicitly on the introduced level distinction, too. Two words that are synonymous, such as “cat” and the German “Katze”, contain tokens of the same meaning, but tokens of different purely graphical (or acoustic) entities. This means, in the terminology introduced, that graphical entities such as [cat] and [Katze] are concrete collections upon which the same linguistic (type) meaning supervenes. A word is more than the sum of its letters and their mutual spatial relations. It is an emergent whole containing (at least) two-levels, and whose supervenient part/property is the meaning of the word.

What smileys are in relation to their concrete shape collections, computer images are in relation to their concrete pixel collections. A pixel (short for “picture element”) is a single-coloured point or square in a graphic image. Each pixel can be represented as having a red-value (R), a green-value (G), a blue-value (B), and three spatial coordinate values: (R, G, B, x, y, z). This representational possibility is due to the fact that each colour can be created as a mixture of some hue of red, some hue of green, and some of blue. The resolution of such an image is the number of pixels on the horizontal axis (a) multiplied by the number of pixels on the vertical axis (b), i.e., a resolution is represented as “a x b” or by the resulting number. Two images that picture the same thing with different resolutions have, by definition, different kinds of concrete collections, since there are then different numbers of members in the two collections.

Where there is a graphic image there is always a concrete collection of pixels, but not vice versa; where there is a concrete collection of pixels there need not be an image. So far so good, but what relevance do these observations have for computer science and informatics? Because in these observations there is hidden a formal inference rule with which it might be useful for computer scientists and informaticians to become familiar. My claim is not completely new. I will develop and make clearer an idea recently put forward in a paper called “Supervenience in Content-Based Image Retrieval” (Ten Brinke et al. 2004).

3 The Supervenience Rule for Emergent Wholes

At first, it might seem as if there cannot possibly be any valid formal inference rule that connects an emergent whole with its concrete collection. By “formal inference rule” I mean a rule that is formal-logical, purely conceptual, or purely mathematical. If there were one, then it would seem that the whole in question could not, because of the very definition of “emergent”, be described as being emergent. And, partly, this is true. If “E” is a description of an emergent whole, and “CC” is a description of some kind of concrete collection that can be a base for E, the following statement – containing *the plain rule for emergent wholes* – cannot be a true statement, and the contained rule cannot be formally valid:

- Necessarily, if there is a case of CC, then there is a case of E.

There can be no formal rule saying that from the existence of a case of the kind CC the existence of a case of kind E can be derived. That is, from merely the description of a concrete collection of pixels one cannot deduce the description of the corresponding image; from merely the description of the concrete collection [:-)] one cannot deduce that there is a smiley; and from merely a description of the letters in a word one cannot deduce the meaning of the word; cf. (Hare 1969: 145). Of course, there can be *non-formal* inference rules stating: if there is a case of CC, then conclude there is a case of E, too. But such rules rely on empirically established correlations, law-like regularities, or natural laws. To find such laws for perceptions were one of the tasks that the Berlin school of Gestalt psychology set itself. (They only partly succeeded; their presumed laws are quite vague. The six main laws are called the laws of, in turn: proximity, similarity, good continuation, closure, good form, and figure/ground.)

The invalid plain rule for emergent properties should be carefully distinguished from a similar formal rule that is valid, and which I will now present. This latter rule is a simple consequence of a distinction one can make between a minimum description of a concrete collection (a “CC^{min}-description”) and descriptions of arbitrary properties of the same concrete collection (“CC^{P}-descriptions”). A CC^{min}-description of a concrete collection of pixels consists of the conjunction of all the statements which ascribe pixels definite values on the variables (R, G, B, x, y, z). CC^P-descriptions, on the other hand, can be single statements such as “This collection contains three hundred red-5 pixels”, “Pixel number 99 is green-4”, “The distance between pixels number 20 and 30 is 0,5cm”, “There are as many blue pixels on the left side as on the right side”, and “This collection of pixels is rectangular”; each singular statement of the minimum description is also a CC^P-description. From the mere definitions of the terms “CC^{min}-description” and “CC^P-descriptions”, it follows that all CC^P-descriptions of a concrete collection can, by means of only formal-logical and mathematical operations (a local spatial coordinate system is taken for granted), be inferred from the minimum description of the same concrete collection. The following statement – containing *the plain rule for concrete collections* – is true, but it does not mention emergent wholes at all:

- Necessarily, if there is a case of CC^{min}, then there is a case of CC^P.

Even though the plain rule for emergent wholes is not formally valid, there is another but more complicated inference rule for emergent wholes that is so valid. For reasons belonging to the history of philosophy (explained in section 4 with reference to R. M. Hare), I will call it *the supervenience rule for emergent wholes*. It is contained in the following statement:

- Necessarily, if a certain concrete collection of kind CC constitutes an emergent whole of kind E, then all other cases of CC constitutes an E, too – *ceteris paribus*.

The *ceteris paribus* clause means “all other relevant things being the same”. Such a clause is needed, since in many common examples of emergent wholes there are hidden parameters whose values are assumed either to be constant or to vary only within a certain range. What this concretely can mean will soon be described, but first some exemplifications of the rule:

(a) Necessarily, if a certain concrete collection of pixels constitutes an image of Västerås, then all other qualitatively identical concrete collections constitute images of Västerås, too – *ceteris paribus*

(b) Necessarily, if this concrete collection [:-)] constitutes a smiley, then all other qualitatively identical concrete collections constitute smileys, too – *ceteris paribus*

(c) Necessarily, if this concrete collection [cat] constitutes a word with the meaning of cat, then all other qualitatively identical concrete collections constitute such a word, too – *ceteris paribus*.

The *ceteris paribus* clauses do here take care of the fact that images, smileys, and meanings, respectively, are mind-dependent, i.e., these entities exist directly only in the eye of a beholder, and this is not explicitly said. Without perceiving minds there are no images and smileys, and without language-using minds there are no meanings. Also, the clauses take care of the fact that there may be a hidden context-dependence as well. For example, the word “cat” has not in all contexts its ordinary meaning; think of the sentence “He let the cat out of the bag”.

However, nothing in the concept of “emergent whole” entails that emergent wholes have to be mind-dependent as in the examples above. Whether or not all emergent wholes are mind-dependent is a truly philosophical issue that one has to enter basic philosophical ontology in order to resolve. If, though, one is a non-reductionist with respect to biological and medical reality (as I am), then there is no essential mind-dependence hidden in the next three examples of supervening emergent wholes:

(d) Necessarily, if a certain concrete collection of molecules constitutes a cell, then all other qualitatively identical concrete collections constitute cells, too

(e) Necessarily, if a certain concrete collection of cells constitutes a liver (an organ), then all other qualitatively identical concrete collections constitute livers (organs), too

(f) Necessarily, if a certain concrete collection of organs, tissues, and substances constitute a cat (organism), then all other qualitatively identical concrete collections constitute cats (organisms), too

These three examples make also another thing clear. There can be hierarchies of on top of each other supervening emergent wholes. Another example of such a hierarchy is to be found in the paper “Supervenience in Content-Based Image Retrieval” mentioned at the end of section 1; here, images are assumed to supervene on collections of “visual features”, which, in turn, supervene on collections of pixels.

4 The Peculiarity of Non-Reductionist Supervenience

The supervenience rule for emergent wholes is formally valid. Here come two examples in which this fact is displayed. It is logically impossible to say that a certain concrete collection of pixels is an image of Västerås and to maintain at the same time that there might have been another similar collection of pixels placed in precisely the same circumstances as the first collection, but which differed from the first in this respect only, that it was not an image of Västerås. Similarly, it is logically impossible to say that this concrete collection of letters [cat] means cat and to maintain at the same time that there might have been another similar concrete collection of letters placed in precisely the same circumstances as the first collection, but which differed from the first in this respect only, that it did not mean cat. My examples are paraphrases of Hare’s (Hare 1969: 145).

Even though the supervenience rule for emergent wholes is formally valid, the plain rule for emergent wholes is not. This is the reason why the supervenience rule is needed. Let me compare with rules for concrete collections. In contradistinction to the plain rule for emergent wholes, the plain rule for concrete collections is formally valid. However, the latter rule entails a supervenience rule for concrete collections. The statement “Necessarily, if there is a case of CC^{min}, then there is a case of CC^{P}” entails the statement “Necessarily, if a certain concrete collection of kind CC has the property CC^{P}, then all other cases of CC has the property CC^{P}, too”. Therefore, there is no need to state a special supervenience rule for concrete collections.

The observations just made can also be summarized as follows: *With respect to emergent wholes there is no plain inference rule, but there is a supervenience rule; with respect to concrete collections there is a plain inference rule and, therefore, a supervenience rule, too.*

The importance of keeping the difference between supervening emergent wholes and properties of concrete collections clear can be seen by means of a comparison of the three assertions below (modelled after (Hare 1984: 2)); all of which describe the same two images and emergent wholes, E_1 and E_2:

(1) E_1 and E_2 consist of the same kind of pixels, E_1 is a *Västerås* image, but E_2 is not a *Västerås* image;
(2) E_1 and E_2 consist of the same kind of pixels, E_1 is an *old* image, but E_2 is not an *old* image;
(3) E_1 and E_2 consist of the same kind of pixels, E_1 is a *rectangular* image, but E_2 is not a *rectangular* image.

Assertion (1) is a self-contradiction that denies the logical truth that contains the supervenience rule for emergent wholes, whereas (2) is an ordinary empirical statement; the latter goes also for the simple statement “E_2 is not a *Västerås* image”. Assertion (3) is a self-contradiction that denies the logical truth that contains the plain rule for concrete collections (and, thereby, denies the entailed supervenience rule for concrete collections, too). To be a Västerås image is for an image to be an emergent whole in relation to a constituting concrete collection; to be an old image is for an image to share a property with its *particular* constituting concrete collection; and to be a rectangular image is for an image to share a CC^{P}-property with its *kind* of concrete collection.

5 The Supervenience Rule for Properties

Supervenient entities can be more or less independent of the properties of the concrete collections that are their base. If the members of the concrete collection [cat] are painted red, whereupon we get the new concrete collection [cat], then this change does not affect the supervening meaning of the word at all. Meanings are neither black nor red. However, if the members of the concrete collection [:-)] are painted red, i.e., we get [:-)], then the supervenient smiling face becomes changed, too. Instead of a black smiley we get a red smiley. When a supervenient entity is quite independent of its base, it seems natural now and then in thought to isolate it from its base and call it a supervenient *property*, whereas when it is not so independent it seems better always to speak only of an emergent whole. In both kinds of cases though, let it be stressed, there is an emergent whole that is *constituted by* a concrete collection plus some kind of supervenient entity. The supervenient entity, be it a property or not, might to be said to be *founded on* the concrete collection in question. In this terminology, the essence of example (c) above can also be captured by saying:

(g) Necessarily, if this concrete collection [cat] founds the meaning of cat, then all other qualitatively identical concrete collections found the meaning of cat, too – *ceteris paribus*.

Where there is a supervenient property, there is an emergent whole. A concrete collection that founds a supervenient property also, thereby, constitutes an emergent whole, namely the whole consisting of this concrete collection plus the founded supervenient property.

The supervenience relation was not, as here, first made explicit in relation to emergent wholes, but in relation to properties. This was done by the moral philosopher R. M. Hare in an attempt of his to become clearer about the relationship between ascriptions of moral goodness and ascriptions of ordinary natural non-evaluative properties (Hare 1969); a brief presentation of the history of the philosophy of supervenience can be found in (Johansson 2001). Hare claimed that the following is true:

(h) Necessarily, if a certain person P is morally good, then every other person that has exactly the same natural properties as P is morally good, too – *ceteris paribus*.

Moral goodness is here looked upon as being a property founded on, but not identical with, natural properties. It is, in other words, a non-reducible supervenient property. The general inference rule for such properties – *the supervenience rule for properties* – is contained in the following statement:

- Necessarily, if a certain concrete collection of kind CC founds a supervenient property of kind S, then all other cases of CC found an S, too – *ceteris paribus*.

If the *ceteris paribus* clause is used to pick out a certain point in time, then the rule becomes applicable also to properties such as being fashionable and being outmoded.

6 Patterns

So far, I have distinguished between (mere) collections, concrete collections, properties of concrete collections (CC^Ps), emergent wholes, and supervenient properties. I will not dwell upon it, but I would like to mention that in-between properties of concrete collections and emergent wholes it seems to be possible to insert even *patterns* as a special kind of entity. Look at the following concrete collections: (a') [- ˙ ˙)] and (b') [:-)]. Both of them, each in its own way, display a colour pattern. A pattern is always a pattern of something that varies, as in (a') and (b') the colour does. In order to perceive or apprehend a pattern, one has to be able to see differences and similarities between all the varying determinates (here: red, green, blue) of the determinable (here: colour) in question. As concrete collections have been defined in this paper, such differences and similarities are neither parts of nor properties of concrete collections. If they were to be incorporated in the definition, this would affect the formal validity of the *plain rule* for concrete collections. A true statement such as “yellow is more like orange than red” seems to be neither a formal-logical, nor a conceptual, and nor a mathematical truth. However, quite independently of whether or not there is a plain formal rule for patterns, there is surely a supervenience rule for patterns:

- Necessarily, if a certain concrete collection of kind CC constitutes a certain pattern P, then all other cases of CC constitute a P, too – *ceteris paribus*.

An analysis of patterns is made in [6], but it contains no discussion of supervenience. Now back, in the very last section, to emergent wholes and supervenient properties.

7 David Lewis’ Unhappy Conflation

As should be clear, I have so far used only Hare’s concept of supervenience. In this section, I will comment on David Lewis’ corresponding concept (Lewis 1986, 1999) (Johansson 2002a). Also, I will now put forward my already promised criticism of the Lewis-inspired paper “Supervenience in Content-Based Image Retrieval” (Ten Brinke et al. 2004).

Both the supervenience rule for emergent wholes and that for properties can be expressed by means of “indiscernibility statements” (see below), and the same goes for the plain rule for concrete collections. To claim that two concrete collections are indiscernible is to claim that they are qualitatively identical. We get, in turn:

(A) Indiscernibility of concrete collections entails indiscernibility of emergent wholes;
(B) Indiscernibility of concrete collections entails indiscernibility of supervenient properties;
(C) Indiscernibility of concrete collections (CC^{min}s) entails indiscernibility of complete sums of properties of concrete collections (sum of CC^{P}s).

Each of the three statements above is logically equivalent with the corresponding statement below:

(A') Necessarily, no difference between emergent wholes without a difference in the corresponding concrete collections;
(B') Necessarily, no difference between supervenient properties without a difference in the corresponding concrete collections;
(C') Necessarily, no difference between complete sums of CC^{P}s without a difference in the corresponding concrete collections (CC^{min}s).

As is easily seen, one can abstract the differences between A', B', and C' away and say:

- Necessarily, no difference of the first sort without a difference in the second sort (in the concrete collections).

So far, everything is unproblematic. However, if one takes the last necessity statement to supply a definition of supervenience, then one skips over the distinction between on the one hand emergent wholes and (Hare-)supervenient properties, and on the other hand ordinary properties of concrete collections. Thereby, one hides also the fact that in relation to concrete collections, but not in relation to emergent wholes and (Hare-)supervenient properties, there exists a formally valid *plain* inference rule.

Unhappily, David Lewis defines “supervenience” in the way described above. He says: “supervenience means that there *could* be no difference of the one sort without difference of the other sort (Lewis 1986: 15).” He also puts it like this: “To say that so-and-so supervenes on such-and-such is to say that there can be no difference in respect of so-and-so without difference in respect of such-and-such (Lewis 1999: 29).” Thereby (writing after Hare), he conflates what Hare is eager to distinguish. As I have explained in section 3, Hare claims rightly that there is an important difference in the way statements such as

(1) E_1 and E_2 consist of the same kind of pixels, E_1 is a *Västerås* image, but E_2 is not a *Västerås* image; and
(3) E_1 and E_2 consist of the same kind of pixels, E_1 is a *rectangular* image, but E_2 is not a *rectangular* image

are self-contradictory. This example, let it be said, is mine; Hare talked of rooms and the properties of "being nice" (which is supervenient) and "being hexahedral" (which is not supervenient) (Hare 1984: 2).

Fact of the matter is that Lewis does not even notice and discuss the issue now highlighted. This is the reason why he seemingly (but falsely) just takes it for granted that if S supervenes on B then it is adequate to say that S has been reduced to, or is "nothing but", B. For Hare, it is other way round: if S supervenes on B, then S cannot possibly be regarded as being reducible to, or being nothing but, B. Lewis can speak in the following way:

> A dot-matrix picture has global properties – it is symmetrical, it is cluttered, and whatnot – and yet all there is to the pictures is dots and non-dots at each point of the matrix. The global properties are nothing but patterns in the dots. They supervene: no two pictures could differ in their global properties without differing, somewhere, in whether there is or isn't a dot (Lewis 1986: 14).

Let me here focus on the property of being *symmetrical*; with respect to *pattern*, I refer to section 5 above. For a dot-matrix picture to be symmetrical is for it to have more or less the same kind of properties on the left side as on the right side. Now, as I have defined "concrete collection", "CC^{min}-description", "CC^P-descriptions", and "formal inference rule", the property of being symmetrical (CC^P) can with the help of the plain rule for concrete collections be directly derived from the CC^{min}-description in question. One picture is enough, whereas in cases of supervenience at least two are required. No talk of supervenience – in either Hare's or Lewis' sense – is here needed. This is the oversight of Lewis.

David Lewis has been quite influential in the philosophy of supervenience, and it is no wonder that the authors of "Supervenience in Content-Based Image Retrieval" (CBIR) rely on him. Nonethelss, they mention in passing the existence also of a non-reductionist notion of supervenience; not Hare's though, but Jaegwon Kim's, which I discuss and relate to Hare in (Johansson 2001, 2002b). Their reason for choosing Lewis' conception is:

> We stick to Lewis' rather than Kim's definition, because in CBIR we do want the visual features to be computable from the pixels. That is not to say that we want 'reversibility', on the contrary, we want to lose details that are not useful for human vision. So, in CBIR we do not want to restrict supervenience to cases of non-reducibility (Ten Brinke et al. 2004: 303).

I would like to say as follows. What is computable or not cannot be a matter of definition of the concept of supervenience. On my (Harean) account of superveneince, both non-reducible and reducible cases are as computable as they are on Lewis' account. In cases of non-reducibility, one of the supervenience rules presented has to be used; in cases of so-called reducibility, the plain rule for concrete collections is preferably used. Those who choose Lewis' definition of supervenience will probably, in some computability contexts, have to use an unnecessary complicated inference rule.

As far as I can see, Lewis' definition of supervenience has no advantages. In philosophical ontology, it obliterates the distinction between emergent and non-emergent entities, and in informatics and computer science it may complicate computing.

References

Hare, R. M. (1969) The Language of Morals. London. Oxford University Press.
Hare, R. M. (1984) Supervenience. In: Aristotelian Society Supplementary Volumes. Vol. 58. pp. 1-16.
Johansson, Ingvar (1998) Pattern as an Ontological Category. In: Guarino Nicola (Ed.) (1998) Formal Ontology in Information Systems (FOIS 1998). Amsterdam. IOS Press. pp. 86-94.
Johansson, Ingvar (2001) Hartmann's Nonreductive Materialism, Superimposition, and Supervenience. In: Axiomathes. An International Journal in Ontology and Cognitive Systems. Vol. 12. pp. 195-215.
Johansson, Ingvar (2002a) Critical Notice of Armstrong's and Lewis' Concepts of Supervenience. In: SATS – Nordic Journal of Philosophy. Vol. 3. pp. 119-122.
Johansson, Ingvar (2002b) Hare's Supervenience Revisited and Re-explicated. http://hem.passagen.se/ijohansson/index.html
Lewis, David (1986) On the Plurality of Worlds. Oxford. Basil Blackwell.
Lewis David (1999) Papers in Metaphysics and Epistemology. Cambridge. Cambridge University Press.