Constitution as a Relation within Mathematics

Constitution is a relation that might be said to be situated between identity and separate existence. If *a* constitutes *b*, then *a* and *b* are separate entities, even though they are located in the same place, and *b* may not be able to exist in separation from *a*. Normally, when the existence of the relation is discussed, it is discussed as being a possible or impossible relation between spatiotemporally located individual entities. This state of affairs is reflected in the fact that the 2012 version of Stanford's online encyclopedia has no entry simply called 'Constitution', only one called 'Material Constitution' (Wasserman 2009). In what follows, I will take it for granted that, really, there is a synchronic relation of material constitution that saves our everyday world from ontological reduction; a relation very much like the one argued for by Lynne Rudder Baker (*Persons and Bodies* [2000] and *The Metaphysics of Everyday Life* [2007]) and Amie L. Thomasson (*Ordinary Objects* [2007]). What I will argue is that there is a relation of constitution within the realm of mathematical entities, too. For instance, I will claim that the mathematical points of a mathematical line are to the line what (to take the most common example of material constitution) the matter of a statue is to the statue.[1] The argumentation is intended to be of such a character that it can be followed, at least in principle, even by philosophers who have some knowledge of elementary mathematics but no special training in mathematics.

The strong thesis of this paper, then, is that there are constitution relations within mathematics. Hopefully, however, readers who are not convinced may nonetheless accept that there is as much a problem around the relation of constitution within mathematics, as there is one within the realm of everyday life.

In order to avoid all misunderstandings, let me at once say that I do not think that mathematicians as mathematicians have anything to learn from what I am saying, but this is no stranger than the fact that in everyday

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life people can talk about and discuss statues and their matter without any problems. In both cases, the constitution problem is a philosophical problem that is importantly related to the issue of ontological reduction versus nonreduction. In the last section, I will make some comments on the relationship between mathematical constitution and a neglected kind of material constitution, property constitution.

1. Material Constitution

What are the typical features of the synchronic relation of material constitution? For the purpose of this paper, i.e., to discuss mathematical constitution, I think it is enough to list three necessary conditions for constitution to obtain (i, ii, iii), and to add two remarks (iv, v).[2] I will allow myself to use modal terminology freely, and the sentences '*a* constitutes *b*' and '*b* is constituted by *a*' are understood as describing the very same state of affairs.

(i) The Colocation Requirement (CR): *if a constitutes b, then a and b coincide spatially.* A statue (*b*) is where its matter (*a*) is, and a person (*b*) is where the body (*a*) is. CR is only a necessary condition; there are states of affairs that may lay claim to consist of two spatially coinciding entities, but where there is no constitution relation. If the classical Faraday-Maxwell wave theory of electromagnetic radiation is true, and can be given a realist interpretation, then many electromagnetic waves can, without losing their identity, exist in the same spatial region simultaneously; the phenomenon is called superposition.

(ii) The Different Identities Requirement (DIR): *if a constitutes b, then a and b cannot be qualitatively identical.* A statue is a different kind of entity than the kind of matter it is made of; and a person is a different kind of entity than a purely biological human organism is. Because of DIR, I will let the terms '*a*' and '*b*' refer to spatiotemporal individuals of kinds A and B, respectively; that is, '*a*' means *this case of A*, and '*b*' means *this case of B*. Nominalists can try to turn everything I say into the nominalist mode of speech by exchanging 'case of X' for 'falling under the predicate 'X', and exchange the term 'constitution relation' for 'falling under the relational predicate 'constitution'.

Since two entities that are qualitatively different cannot be numerically identical, DIR entails (what is taken for granted from the start) that *a* and *b* are numerically different entities. DIR must, however, be stated in terms of qualitative identity, since, conversely, two numerically different entities may be qualitatively identical; I do not regard a relational property such as spatial position as belonging to an entity's qualitative identity. Normally (if not always), material entities that have different kinds of identities have different persistence conditions and different modal properties, too. A clay statue may disappear but the aggregate of clay particles stay the same aggregate despite a radical change in outline shape. Shape is an essential property of a statue, but not of a collection of clay particles. Such a lump has other essential properties: consisting of certain kinds of particles, having a certain volume, and having a certain mass.[3]

When I say that *a* and *b* must not be qualitatively identical, I mean that they have to be either different kinds of entities or different properties. Baker and Thomasson restrict constitution to kinds, but I will in the last section show that constitution can make good sense even in relation to some properties.

DIR entails that constitution is an irreflexive relation. Nothing can be constituted by itself. Necessarily, the sentence '*a* constitutes *a*' is false.

Also, DIR of course entails that if *a* constitutes *b*, then *a* and *b* may have different causal powers. I see no reason to claim that, necessarily, *a* and *b* have different causal powers, since I think that existence should not be identified with having causal powers. I regard epiphenomena as being ontologically possible entities.

(iii) The Asymmetry Requirement (AR): *if a constitutes b, then b cannot constitute a*, i.e., constitution is an asymmetric relation. A lump of matter cannot be constituted by a statue, and a biological human organism cannot be constituted by a person. AR is only a necessary requirement, since there are many asymmetric relations (e.g., *larger than* and *brighter than*) that have nothing whatsoever to do with constitution. Often, the asymmetry intuition seems to be based in a relation of one-sided existential dependence. That is, if *a* is taken away (and not substituted by something else that functions as a constituting part) then *b* cannot exist, whereas *a* may continue to exist even if *b* does not.[4]

(iv) Some philosophers seem to regard the constitution relation as a transitive relation (Simons 1987, 237f), whereas others definitely take it to be nontransitive (Baker 2000, 44–46; 2007, ch. 9). Probably, this difference has to do with an ambiguity in the term 'constitution'. It is not made clear whether the term is understood in the narrow sense of *direct* constitution or in a wider sense. If a directly constitutes b, then there are no entities in-between a and b that can claim to constitute b. By definition, direct constitution is intransitive. It is easily seen in relation to social entities. If c is an organization (directly) constituted by a number of other organizations ($b_1 - b_n$), and each of these organizations are (directly) constituted by a number of persons ($a_1 - a_m$), then c cannot be (directly) constituted by the persons mentioned.

On the other hand, if also indirect constitution is counted as constitution, then, equally trivially, constitution becomes a transitive relation.

Mereology, in all its contemporary versions, contains a transitivity axiom. This fact makes Baker maintain that constitution is not at all a part/whole relation (Baker 2000, 44; 2007, 32 and 181f); even though she allows people using everyday language to talk in the contrary way. I think, however, that mereology should not even in philosophy have monopoly on the term 'parthood'; and I will now and then below use the term 'constitutive part'.[5]

(v) Normally, when a case of kind A constitutes a case of kind B, then it is not a only as A (the essential properties of a) that constitutes b as B. It is a as A *plus* some specific contingent properties of a. In other words: "When x constitutes y, there are certain properties of x which are *accidental* to x, but *essential* to y. It is essential to a person that he or she have certain processes going on in his/her body, but accidental to the body that these processes be going on in it" (Simons 1987, 239). In the case of the statue and the clay, a particular shape that is contingent to the collection of clay particles is essential to the statue.

2. Perceptual Constitution

Before turning to mathematical constitution, I will say some words about constitution in the perceptual field. In my opinion it is beyond doubt that in many perceptions there are, in the way I will explain, relations of constitution; both in illusory and veridical perceptions. My observations are closely connected with some remarks once made within *Gestalt* psychology, but there is no need to dwell on this point.[6] Important to note, however, is that I make no attempts to put forward any causal explanations. My point is that constitution can be a perceptually given datum;

whatever the causes are. Because of this fact, it cannot possibly be true that the term 'constitution' has no discernible sense and extension at all.

Look at the area within the curled brackets { - ˙ ˙) }. Naturally, because of our perceptual system, it appears as containing a collection of four elements (one straight line, two dots, and one bent line) with some specific spatial relations between them. In {˙ ˙) -} the same collection of elements appears again, but with other spatial relations between them. That is, one and the same collection can exist in many different kinds of spatial configurations. The essence of the collection is that it has four elements, one of which is a straight line of a certain length, two of which are dots, and one of which is a bent line of a certain length. Each specific spatial configuration is a contingent property of the collection mentioned, but it is essential to the collection that at each point of time it has some specific contingent configuration.

Let's now take a look at the collection in the following spatial configuration { :-) }. Here there is not only the collection with a contingent specific spatial configuration; there is in the perceptual field also (to most of us) the well-known computer Smiley. If we regard the collection in its configuration as informally being a sum of the elements and the spatial relations involved, then Smiley is more than this sum. Smiley and the given case of the collection fulfill the necessary requirements for constitution. They coincide spatially in the perceptual field (CR), they are qualitatively different kinds of entities (DIR), and Smiley seems asymmetrically be more dependent on the collection than vice versa (AR). According to the ordinary constitution intuitions, Smiley can be regarded as being constituted by the collection in the contingent spatial configuration at hand.

Now, someone might object that it is simply not true that in the perceptual field there are simultaneously both a Smiley and a collection: *either* we perceive a Smiley *or* we perceive a collection. I think this is false. I find it possible to have a kind of split vision where at one and the same time I see both Smiley and the collection. Smiley is the foreground and the collection is a kind of background; not a background beside the foreground, but in the midst of it. That is, I do *not* regard the difference between the collection and the Smiley as being a case of ambiguous pictures such as the famous duck-rabbit. In ambiguous pictures one can only perceive one of the figures at a time; even here, though, it is possible in a split vision simultaneously to see both the duck and the lines, and in another split vision see both the rabbit and the lines.

I regard the *Gestalt* psychology slogan "the whole is more than the sum of its parts" as an implicit acceptance of a constitution relation. *Gestalten* are emergent wholes constituted by a number of constitutive parts.[7]

3. Mathematical Constitution

If mathematics were to be restricted to investigations of operations on numbers and abstract structures of these, then it might be impossible to find intuitions that lead in the direction of constitution relations. But mathematics contains geometry and topology, too; there are axioms and theorems about mathematical points, lines, surfaces, and bodies situated in an abstract ideal space, often a coordinate system regarded as a vector space. Therefore, it is possible to discuss colocation (CR) also of mathematical entities. In this and the next section, it will be taken for granted that the space in question is an ideal two-dimensional Euclidean space; that is, only mathematical points, lines, and surfaces within a Euclidean framework will be discussed.

Before I continue, perhaps some words about the very existence of mathematical points and lines are needed. Nondivisible points and breadthless lines were explicitly introduced already by Euclid in his *Elements*. However, the development of mathematics and the arithmetization of geometry might be regarded as having had the same kind of deletion effect on mathematical-geometrical points and lines as the development of chemistry has had on phlogiston, and the development of physics has had on the ether, i.e., make them, each in their own way, regarded as being nonexisting. I think this is false. Even though phlogiston and the ether can be deemed chemically and physically nonexisting, points and lines cannot be deemed mathematically nonexisting; let me briefly say why.

The so-called Cantor-Dedekind (or Dedekind-Cantor) axiom says that the points on a line can be put into a *one-to-one correspondence* with the *real numbers*. This axiom allows the algebra of the real numbers to be employed for obtaining results about geometrical continua; relying on it, all the theorems of Euclidean geometry can be given an arithmetic interpretation. But this fact does not make infinitesimally small points and infinitesimally thin lines mathematically nonexisting; rather the contrary.

If they are regarded as nonexisting, then the isomorphism of the Cantor-Dedekind axiom would—very astonishingly—be about nothing at all; for an isomorphism to make sense, there must be two existing kinds of relata.[8]

In order to discuss mathematical lines (both infinitely long and line segments) and points from a constitution point of view, one has to decide, just as in material constitution, what to regard as essential properties and contingent properties, respectively, of the related entities under discussion. Look at the following three lines, and let each of them represent an infinitesimally thin continuous mathematical kind of line in a two-dimensional space: (a) ——, (b) —, (c)) .

The lines (a) and (b) have the same shape, being straight, but they are different kinds of lines because they are of different lengths. The lines (b) and (c) should be regarded as having the same length, but they are different kinds of lines because they have different shapes, straight and bent, respectively. As I am going to use the term '(mathematical) line', each *kind* of line has three essential properties: *infinitesimal thinness*, a *specific length* (be it finite or infinite), and a *specific shape* (be it closed or open). The same kind of line can be in several places in the same abstract space; the introduced distinction between numerical and qualitative identity is as applicable to mathematical lines in an abstract space as it is to properties of material things in real space.

All mathematical lines do in some sense contain a collection of mathematical points, but a collection of points need not be ordered along a line. I will define *kinds* of point collections by means of two essential properties: all the elements have *infinitesimal extent*, and the collection contains a *specific number of elements* called 'the size of the collection'. The size can be that of any finite natural number or that of a transfinite cardinal number. In the three cases (a), (b), and (c), the size is the same, namely the cardinality of the set of the real numbers; every finite continuous line has as many points as the real number line has. That is, the three kinds of lines mentioned contain the same *kind* of collection. It is a contingent property of the collection how its points are spatially related to each other, and where in space they are situated.

The situation described is analogous to the one where we have three kinds of statues that in principle could have been made of the same collection of clay particles. Allowing some informal topology, there is also an analogy between the mathematical point collection and a lump of clay that consecutively is turned into three different kinds of statues. Topology studies properties of geometrical objects that are preserved under continuous deformations. And in this sense the point collection contained in line (a) can without losing its identity be compressed into the configuration making up line (b), which, in turn, without losing its identity, can be bent into the line (c).

In each of the cases (a), (b), and (c), the necessary requirements for constitution are met. There is colocation of a line and a contingent property of a point collection (CR), the line and the point collection are different kinds of entities (DIR), and there is asymmetry, the line depends on the point collection, but not vice versa (AR). The line is also "more than the sum of its constitutive parts," since a sum of nonextended entities cannot be an extended entity. So, by analogy: if there is a constitution relation between a clay statue and the clay particle collection (the lump) it contains, then there is a constitution relation between a mathematical line and the point collection it contains.

The mathematical constitution view put forward affects how to look upon classical analytic geometry. For instance, a straight line in the x-y plane should not be regarded as being *identical* with the point collection that consists of all the points that (between two specific points) satisfy an equation of the form $y = a + kx$; the line should be regarded as *constituted* by such a collection. Similarly, a circle with a radius r around the origin should not be regarded as being identical with a point collection consisting of all the points that satisfy an equation of the form $x^2 + y^2 = r^2$; it should be regarded as being constituted by the collection. And so on for every possible line. Arithmetic formulas such as '$y = a + kx$' and '$x^2 + y^2 = r^2$' become ambiguous in relation to nonanalytic geometry. Out of context, they can represent both a line and a point collection.

Since the length of a line can be calculated by means of integration over a function $y = f(x)$ that represents a line, the view put forward also affects how to look upon certain operations of integration. The length of a finite line cannot possibly be identical with an arithmetic sum of its infinite number of contained infinitesimal points; such a sum must give us either a line of zero length or one of infinite length. If the points are assumed to have no extension at all, then the arithmetic sum cannot possibly be ascribed any extension either; and if the points are assumed to have an extension, however small, then the length of the line has to be infinitely long. The integration operation is the topic of the next section, even though for pedagogical reasons I will not discuss it when going from points to a line, but when going from lines to a surface; as is usual in introductions of the Riemann integral. By analogy, we can then say that what a collection of mathematical lines is to the surface that contains them a collection of mathematical points is to the line that contains them.

It should be noted from the start that the meanings of the very terms 'to integrate' (i.e., to put together parts and combine them into a whole) and 'to constitute' (i.e., to establish some kind of unit) are quite similar.

4. Integration as Representing Mathematical Constitution

As addition of numbers comes with the inverse operation of subtraction, and multiplication with the inverse of division, integration of functions, $\int f(x)\, dx$, comes with the inverse operation of differentiation. If the integration of the one-variable function $f(x)$ yields $F(x)$, then the differentiation of $F(x)$ yields $f(x)$, but I will discuss only integration. When there is an integration between two specific numbers a and b of the variable x, then it holds true: $\int_a^b f(x)\, dx = F(a) - F(b)$. When $f(x) = x^n$, then there is a general algorithm that says that $F(x) = x^{n+1}/(n+1)$; for instance, when $f(x) = x^2$, then $F(x) = x^3/3$. I will argue that in some cases the operation can be understood as representing a relation of constitution. Out of the entities represented by x and y in $y = f(x)$ the integration operation constitutes the entity represented by $F(a) - F(b)$. The integration operation can be defined also for many variables; it then gives rise to double integrals $\iint f(x, y)$, triple integrals $\iiint f(x, y, z)$, and so on for other multiple integrals.

Look at Figure 1 below. The area of each of the two times five strips (the shorter contained in the taller) can be calculated as the height of the strip multiplied by Δx. It is easily seen that that the area between the curve $y = f(x)$ and the points a and b on the x-axis must have a magnitude that lies in-between the sum of the areas of the five taller strips and the sum of the five corresponding shorter ones, whatever unit is used as measurement unit. Intuitively, it is equally clear that if instead of five strips we insert ten strips between a and b (Figure 2), then the magnitude of the area under the curve must still lie in-between the two sums in question; even though the difference between the sums is now smaller. The more strips we insert between a and b, the smaller the difference between the sums will be, but the value of the area magnitude must still be somewhere in-between the values of these sums; call them '$\sum$ upper-F_n' and '$\sum$ lower-F_n', respectively.

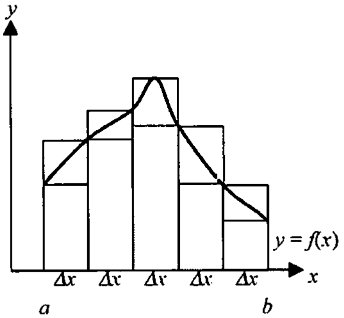

Figure 1

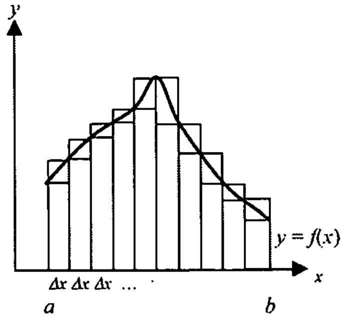

Figure 2

Let me next bring in the notions of 'approaching infinitely large numbers' (= $n \to +\infty$) and 'limit value' by, as is usual, talking about arbitrarily small (ε) and arbitrarily large (ω) numbers:

- The sum $\sum F_n$ has the limit value A when $n \to +\infty$, if to every number $\varepsilon > 0$ there is a number ω, such that: $|\sum F_n - A| < \varepsilon$ for all n such that $n > \omega$.[9]

- The function f has the limit value A, when $x \to +\infty$, if to every number $\varepsilon > 0$ there is a number ω, such that: $|f(x) - A| < \varepsilon$ for all n such that $x > \omega$.

For continuous curves it can be mathematically proven that when n in the construction mentioned approaches infinity, and the width of the strips (Δx) approaches zero, then the difference $\sum$ upper-F_n – $\sum$ lower-F_n approaches zero, since the two sums approach the same limit value, A. This limit value must be regarded as representing the magnitude of the area under the curve. And it is this value that the mathematical integration operation supplies us with: $\int_a^b f(x)\,dx = A$.[10]

In the mathematical procedure presented, and illustrated by Figures 1 and 2, three mathematical processes run parallel. When (i) n approaches infinitely large numbers, then (ii) Δx_n approaches infinitely small numbers, and (iii) $\sum F_n$ approaches a specific finite value, A, the value of the integration operation.

A finite area cannot possibly be regarded as being identical with an arithmetic sum of its infinite number of contained infinitesimal lines (strips);

such a sum gives us either an area of zero size or of an infinite size. If the width of the strips is assumed to be zero, then each strip is only a line and cannot possibly have an area that is an additive part of the whole area; and if the strips are assumed to have a certain width, however small, then the sum of the strip areas becomes infinitely large because of the infinite number of strips. However, thanks to the ingenious operation of integration we can give a well-defined sense to the intuition that an area in some sense can be regarded as containing an infinite number of infinitesimally thin lines. We cannot regard the area as a sum of such lines, but we can regard it as *constituted* by them. The area and the contained lines are co-located (CR), the area and the line collection are different kinds of entities (DIR), and there is asymmetry, the area depends on the line collection, but not vice versa (AR); also, the area "is more than the sum of its constitutive infinitesimal line parts."

5. Mathematical Constitution as Representing Property Constitution

By means of integration it is in the way shown possible to move from $(n - 1)$ to n dimensions in an abstract space, and to regard an n-dimensional entity as being constituted by the $(n - 1)$-dimensional entities it contains. The claim made is so far wholly confined to pure mathematics, but with the help of mathematical physics it can easily be extended to relationships between certain kinds of properties.[11] To start with, let us look at the relationship between mass as a property of three-dimensional material entities, and mass density as a property ascribed to infinitesimal points.

If a completely homogenous particle or field of volume V has the mass m, then each infinitesimal point of the body has the density represented by the ratio $\rho = m/V$. Conversely, the mass can be arrived at from ρ only by means of multiplication of the volume, V; not by addition of the point densities. If the density varies according to a function $\rho = \rho(x, y, z)$, then in order to calculate m one has to use triple integration: $m = \iiint \rho(x, y, z)$. In the light of the former section, this means that mass should not be regarded as identical with a collection of mass densities, but as *constituted* by such a collection.

To a realist, the acceptance or nonacceptance of a constitution relation between mass density and mass makes quite a difference. If such a relation is not accepted, then the realist is faced with a dilemma: should mass be regarded as a really existing property and mass density as a mere instrumental calculation device, or vice versa? It seems to me as if many philosophers who face this hard dilemma choose to dissolve it by saying: *both* the seeming properties must be regarded as only calculation devices. However, if constitution is accepted, then a new option appears: *both* the term 'mass' and the term 'mass density' are able to have real spatiotemporally located property instances as referents.[12]

What has been said about mass density in relation to mass can equally well be said of electric charge density in relation to electric charge and of energy density in relation to energy. In fact, every kind of property that is ascribed to points in space-time, and that can be represented by an integrable function, can in principle be regarded as constituting an entity that is represented by the integral in question. In other words, all field theories may in principle be regarded as implicit bearers of a constitution relation.

Let me summarize this paper.[13] The concept of constitution cannot possibly be regarded as lacking extension altogether; this is shown by the phenomenon of perceptual constitution. For reasons not stated in the paper, I am convinced that there is material constitution. Now, if there is, then by analogy there is also mathematical constitution. And if there is mathematical constitution, then mathematical physics realistically interpreted tells us that there is property constitution, too. I think constitution is ubiquitous.

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NOTES

1. Let me add, that I agree with Randall Dipert who remarked that he has "a minor quibble" with the matter-and-statue discussion. It goes on as if there are no statues that have holes as important parts. I think, however, that there is a comfortable way out. One should acknowledge that there is a "general [ontological] dependence of a hole on its host" (Casati and Varzi 1995, 192), and then say that, for instance, a doughnut-shaped statue of clay is constituted by the clay *and* the hole the clay hosts. In fact, holes might be relevant to mathematical constitution, too. In topology, holes are important. What separates one topological shape from another is their number of holes; a topological shape is the class of ordinary shapes that can be transformed into one another without tearing or ripping. I will not, however, discuss entities with holes in this paper.
2. For an actual definition proposal, see Baker (2000, 39–46). On the surface, it may look as if the definition conforms to first-order predicate logic, but it does not; the variable D for the undefined notion of 'circumstances' must be a variable for properties.
3. I write as if a lump of clay can be identified with a collection of clay particles, but it may well be argued that a lump of clay should be regarded as *constituted* by such a collection. My point in talking of a collection of clay particles instead of simply a lump of clay is to make the analogy with collections of mathematical points more conspicuous.

4. I think there is much more to say about nonsymmetric relations than is usually taken for granted; see Johansson (2011).

5. For an elaborate defense of my allowance of intransitive parthood relations, see Johansson (2008). My central claim there is that expressions such as 'large part', which are nontransitive, have an implicit reference to a third term. In a sentence such as 'x is a large part of y' there is a hidden variable, z. No doubt, all truly two-term parthood relations are transitive.

6. For an introduction to *Gestalt* psychology, see Smith (1988); the book also contains other papers about this way of thinking, as well as papers by some famous *Gestalt* psychologists.

7. For more details about the relationships between emergent wholes and constituting collections, see Johansson (2006). This paper also contains a more elaborated discussion of the phrase 'is more than the sum of its parts'.

8. I am using the term 'infinitesimal' without bothering about whether its extension should be understood as (i) necessarily being only limits of something, as (ii) being amenable to Robinsonian nonstandard analysis, or as (iii) being amenable to so-called smooth infinitesimal analysis; see Bell (2009). I think the constitution problem arises in all three cases.

9. The expressions $|\sum \mathrm{F}_n - A|$ and $|f(x) - A|$ mean that the subtraction will yield a positive value even when A is larger than $\sum \mathrm{F}_n$ or $f(x)$, respectively.

10. Wikipedia's entry "Integral" says (2012): "The discrete equivalent of integration is summation." I think it had better be phrased: "The discrete equivalent of integration is summation after multiplication." Integration should be regarded as an operation in which summation and multiplication make up a fused whole where the two operations are not distinguishable. The expression '$\int f(x)\, dx$' does not symbolize two consecutive operations, first multiplying, '$f(x)\, dx$', and then summing, '$\int$'; it symbolizes one single operation.

11. My general view of the relationship between mathematics and mathematical physics can be found in Johansson (2009).

12. This does not entail that one has to accept that a point density can have an independent physical existence. With Brentano (1976) one can (and I think: should) subscribe to the view that, necessarily, physical point entities exist only as parts of extended physical entities.

13. I think the Gothenburg philosopher Ivar Segelberg (1914–87) was close to the main idea of this paper. However, if for no other reason, his lack of mathematical knowledge stopped him from expounding it; he would, though, have called constituted wholes 'complex unities'. See Segelberg (1999, in particular *Zeno's Paradoxes* ch. III and *Properties* ch. V). My idea was first presented in fall 2010 in a working seminar on the philosophy of science in Lund (Bengt Hansson), then during The Swedish Philosophy Days in Gothenburg, summer 2011, and lastly, fall 2011, in a seminar in theoretical philosophy in Lund (Erik J. Olsson). In all three cases, as in relation to Randall Dipert, I am grateful for critical comments that forced me to pedagogical improvements.

References

Baker, Lynne Rudder 2000. *Persons and Bodies: A Constitution View*, Cambridge: Cambridge University Press.

———. 2007. *The Metaphysics of Everyday Life: An Essay in Practical Realism*, Cambridge: Cambridge University Press.

Bell, John L. 2009. "Continuity and Infinitesimals," in *Stanford Encyclopedia of Philosophy*, http://plato.stanford edu/entries/continuity/.

Brentano, Franz 1988. *Philosophical Investigations on Space, Time and the Continuum*, translated from the German ([1976]) by B. Smith, London: Croom Helm.

Casati, Roberto and Achille C. Varzi 1995. *Holes and Other Superficialities*, Cambridge MA: The MIT Press.

Johansson, Ingvar 2006. "Inference Rules, Emergent Wholes and Supervenient Properties," *tripleC—Cognition, Communication, Co-operation* 4(2): 127–35.

———. 2008. "Natural Science and Mereology," in *Handbook of Mereology* (forthcoming), edited by H. Burkhardt, G. Imaguire, and J. Seibt. (See http://hem.passagen.se/ ijohansson/.

———. 2009. "Mathematical Vectors and Physical Vectors," *Dialectica* 63: 433–47.

———. 2011. "Order, Direction, Logical Priority and Ontological Categories," in *Ontological Categories*, edited by J. Cumpa and E. Tegtmeier, Frankfurt: Ontos Verlag, 89–107.

Segelberg, Ivar 1999. *Three Essays in Phenomenology and Ontology*, translated from Swedish ([1945], [1947], [1953]) by H. and S.R. Hochberg, Stockholm: Thales.

Simons, Peter M. 1987. *Parts: A Study in Ontology*, Oxford: Clarendon Press.

Smith, Barry 1988. "Gestalt Theory: An Essay in Philosophy," in *Foundations of Gestalt Theory*, edited by B. Smith, München: Philosophia Verlag, 11–81.

Thomasson, Amie L. 2007. *Ordinary Objects*, Oxford: Oxford University Press.

Wasserman, Ryan 2009. "Material Constitution," in *Stanford Encyclopedia of Philosophy*, http://plato.stanford.edu/entries/material-constitution/.