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# Mechanism Models as Necessary Truths

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**Abstract** The paper argues that there is a fruitful analogy to be made between classic pre-analytic Euclidean geometry and a certain kind of mechanism models, called *ideal mechanisms*. Both supply necessary truths. Bunge is of the opinion that pure mathematics is about fictions, but that mathematics nonetheless is useful in science and technology because we can go "to reality through fictions." Similarly, the paper claims that ideal mechanisms are useful because we can go to real mechanisms through the fictions of ideal mechanisms. The view put forward takes it for granted that two important distinctions concerned with the classification of fictions can be made. One is between ideal and non-ideal fictions, and the other between social and non-social fictions. Pure numbers, purely geometric figures, and ideal mechanisms are claimed to be ideal and social fictions.

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(Figures at the end of the paper)

Since decades, I have been amazed by the way many seemingly erudite philosophers of science completely neglect or are completely unaware of the views of Mario Bunge. Once in 2014 and once in 2015, however, I became truly chocked by this negligence. I had optimistically thought that as soon as mechanisms would become discussed, then, surely, Bunge's views and reflections must be considered by those interested. But no. Neither in the overview article "A Field Guide to Mechanisms" (Andersen 2014) nor in *Stanford Encyclopedia of Philosophy*'s entry "Mechanisms in Science" (Craver & Tabery 2015) is Bunge's name mentioned. That is, to take all doubts away, not mentioned at all! Leaving this highly remarkable fact as a future case study for the sociology of philosophy to explain, let me present his and my thoughts about mechanisms.

## Bunge on Mechanisms and Models of Mechanisms

From the paper "Phenomenological Theories" (Bunge 1964) and onwards, Bunge has stressed a distinction between black box theories and representational translucid box theories. The essential difference between them is located in the concept of *mechanism*: "In other words, a 'mechanism' linking *I* [input data] to *O* [output data] is wanted in the translucid box approach" (Bunge 1964, p.239).[2] Nowadays, Bunge defines mechanisms as follows:

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[2] Apart from (Bunge 1964), see also in particular (Bunge 1997), (Bunge 2004), and (Bunge 2006, ch.5). In a "Personal Postscript" in (Bunge 1997), he presents "a brief account of my struggle with the concepts of

*Definition 1.5* A *mechanism* is a set of processes in a system, such that they bring about or prevent some change – either the emergence of a property or another process – in the system as a whole. (Bunge 2003, p.20)

Two of his claims about mechanisms are: "There is no method, let alone logic, for conjecturing mechanisms" and "the covering-law model [of explanation] fails to capture the concept of explanation used in the sciences, because it does not involve the notion of a mechanism" (Bunge 2006, pp.138 and 139). I agree.

Mechanisms must be kept distinct from *models* of mechanisms. "Mechanisms are processes in concrete (material) systems," but models are "conceptual and semiotic systems" (Bunge 2006, p.129)[3] – and:

The simplest sketch or model of a material system σ is the list of its composition, environment, structure, and mechanism, or

$$\mu(\sigma) = \langle C(\sigma), E(\sigma), S(\sigma), M(\sigma)\rangle.$$

Here, $C(\sigma)$ denotes the set of parts of σ; $E(\sigma)$ the collection of environmental items that act on σ or are acted upon by σ; $S(\sigma)$ the structure, or set of bonds or ties that hold the components of σ together, as well as those that link it to its environment; and $M(\sigma)$ the mechanisms, or characteristic processes, that make σ what it is and the peculiar ways it changes. (Bunge 2006, p.126)

Only one of these four model-elements, $M(\sigma)$, models a mechanism. Bunge has not tried to explicate the notion of $M(\sigma)$ as such. He has rested content with presenting a number of mechanisms put forward in the natural and the social sciences. Late in life, he has also brought in examples from medicine (Bunge 2013).[4] In this paper I will show that the mechanism concept is of such a character that we ought to distinguish between mechanism models that do and do not contain necessary truths, respectively.

This does not mean that I am interested in what exactly can be meant by necessary truths. In contemporary philosophy, the term is often avoided in favor of terms such as (depending on philosopher) 'formal truths', 'analytic truths', 'tautological truths', 'logical truths', 'conceptual truths', 'true in all possible worlds', or something else. What is important to me is the analogy between classic geometry-thinking and the kind of mechanism-thinking that I will present and

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mechanism and mechanismic explanations" (ibid. p.458). For me, (Bunge 1964) was a seminal paper. The notion of mechanism has ever since the end of the 1960s played an important role in my philosophical endeavors. In (Johansson 2004 [1989], ch.14) it is central. Subchapter 14.3 has the heading "Mechanisms and their parts." The views about mechanisms stressed in the present paper were first outlined in (Johansson 1997).

[3] In many contexts the expression 'material model' is used, but, as far as I know, Bunge uses it only on one occasion (Bunge 1967, p.146). I take him to mean that material systems become material models only when being part of "conceptual and semiotic systems."

[4] A co-author and I have, partly influenced by Bunge (see footnote 2), ever since the 1990s stressed the importance of distinguishing in medicine between *mechanism knowledge* and *correlation knowledge*. See (Johansson & Lynøe 2008, ch.6); we did earlier put forward the distinction in similar but much smaller books in Swedish in 1992 and 1997, and in Danish 1999.

defend. By Euclidean geometry I mean geometry as it was before Descartes invented the coordinate system. By means of this, large parts of geometry became amenable to arithmetic and algebraic treatment. That is, analytic geometry came into being; but I will be concerned only with classic pre-analytic geometry.

By the expression 'necessary truths' I mean statements that are true in the same sense – whatever that is – in which Euclid's five Postulates and five Commons Notions (his axioms) except the fifth postulate (the so-called parallel axiom) are still regarded as in some sense being necessary truths (Euclid 2017, p.2). When from such axioms theorems are validly deduced, the theorems are of course necessary truths, too.[5]

Let me take two examples, and at the same time (in contradistinction to Euclid) make the necessity claim explicit. Euclid's first Postulate says that, necessarily, between any two points there is a straight line, and his first Common Notion says that, necessarily, things which are equal to the same thing are also equal to one another. It is statements like these that I claim my forthcoming proposed mechanism axioms are analogous to. From a subjective point of view, a necessary truth says that this is the way the things talked about have to be; they cannot possibly be different.

When using the term 'proposition', I will use it in Bunge's sense of a man-made construct and non-Platonic entity (Bunge 1974b, pp.85-86). A necessary truth can then be represented by the form 'it is true that, necessarily, *p*', where *p* is a variable for propositions. Factual (contingent) truths can be given several forms. I will use only the strictly singular and the strictly universal, i.e., 'it is true that, at time t and place x, *p*' and 'it is true that, always, *p*', respectively.

**The Cogwheel Mechanism**

My first example of a mechanism is the cogwheel mechanism. In models of systems that have cogwheels as their characteristic mechanism, $C(\sigma)$ denotes a number of cogwheels with their axles, $E(\sigma)$ denotes something that makes at least one of the cogwheels rotate, and $S(\sigma)$ denotes the structure which keep the wheels in place and somewhere makes the cogs of one wheel fit into at least one other. But what does the very mechanism model $M(\sigma)$ denote? According to Bunge, it denotes "a set of processes in a system, such that they bring about or prevent some change."

Let us think of a mechanism consisting of two connected circular cogwheels whose axles are paralleled (Fig. 1).

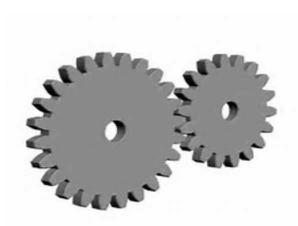

Fig. 1

[5] What today are called theorems, Euclid calls *propositions*. In my discussion I disregard the fact that Euclid's first three Postulates are not statements that describe a state of affairs as existing; they state that it is always possible to *draw*, *produce*, and *describe*, respectively, a certain kind of geometric figure.

When the wheels are equally large, I will label it *the simple cogwheel mechanism*, and call the two wheels $W_1$ and $W_2$, respectively. When nothing else is said, it is taken for granted that something external to $W_1$ makes $W_1$ rotate, and that $W_2$ moves because of $W_1$. One single rotating cogwheel cannot be a mechanism; it constitutes merely a process. As Bunge says: "Every mechanism is a process, but the converse is false" (1997, p.416). A possible practical function of this mechanism is to change rotation direction from one axle to another.

As just indicated, when the wheels in the simple cogwheel mechanism rotate, they rotate in opposite directions. The question I will discuss is whether or not this is a necessary truth. Since, normally, the mechanism in question is part of a material system, the first-hand answer seems to be negative. Why? Because no descriptions of a material system can contain necessary truths. But let us take a closer look at the issue; to start with, let us memorize these two factual hypotheses:

> *The singular hypothesis $H_S$*: At place x and time t, when in the simple cogwheel mechanism wheel $W_1$ makes a single turn in one direction, wheel $W_2$ makes a single turn in the other direction.
>
> *The universal hypothesis $H_U$*: Always, when in the simple cogwheel mechanism wheel $W_1$ makes a single turn in one direction, wheel $W_2$ makes a single turn in the other direction.

So far, nothing has been said about the material of the cogwheels. It has been abstracted away, even though all real cogwheels must be made of some kind of material and have a mass. Now, assume as a first specified case, that wheel $W_1$ is made of steel and $W_2$ of fragile glass. In all probability, if $W_1$ starts to rotate, it will crush a number of cogs on $W_2$, and when $W_1$ has made a whole turn, $W_2$ has not. That is, $H_S$ is in this case false, and the universal hypothesis $H_U$ is falsified.

Secondly, assume that the glass wheel ($W_2$) is exchanged for a very soft wool wheel. This time the cogs of $W_2$ will not be crushed, only bent. However, the result is similar to the glass case. When $W_1$ has made a whole turn, $W_2$ has in all probability not. Therefore, even in this case $H_S$ is false and $H_U$ falsified. Moreover, there are with such wheels even more conspicuous ways of falsifying $H_U$. Assume that the wheels $W_1$ and $W_2$ are by different force sources made to rotate in the *same* direction. Is this possible? Yes, since the steel cogs will bend the wool cogs.

Thirdly, assume that both the wheels are made of steel. Now, at last, $H_S$ (with properly chosen values for *x* and *t*) becomes true. However, we are not allowed to extrapolate and claim that $H_U$ is true as well, which would falsely mean that the process could go on for an infinite number of rotations. In the long run, due to wear and tear, the mechanism will start to malfunction and not rotate properly; and in the very long run, due to rust and other kinds of decay, it will stop function at all. That is, $H_U$ will in the end become falsified even by the pure steel wheel mechanism.

Assume, fourthly, that both the cogwheels are *absolutely rigid bodies*, i.e., they are bodies that:

(i) cannot possibly change shape
(ii) cannot possibly change size
(iii) cannot possibly lose parts or take up new parts
(iv) cannot possibly be destroyed.

By definition, no parts of such wheels can be broken (like the cogs of the glass wheel) or bent (like cogs of the wool wheel), and the mechanism process is not susceptible to wear and tear (as steel wheels are). In fact, absolutely rigid bodies are analogous to the absolutely non-changing two-dimensional figures (triangles, squares, circles, etc.) and three-dimensional solids (pyramids, cubes, spheres, etc.) of Euclidean geometry. Often, such Euclidean objects are called *ideal figures* (plane geometry) and *ideal solids* (solid geometry), respectively; and I will follow suit.

The essential difference between the notions of *ideal solid* and *absolutely rigid body* is that the former notion does not bring in temporal duration, whereas duration is essential to the definition of the latter. To be rigid is defined as not being changeable during a certain time period. Euclidean solids are *atemporal* objects, while rigid bodies are *temporal* objects. An absolutely rigid body is a solid that, necessarily, endures without changing anything else than spatial position. I will also call it an *ideal body*. An ideal figure has a two-dimensional shape, an ideal solid has a three-dimensional spatial shape, but an ideal body has a shape that is extended not only in three spatial dimensions but also in a temporal dimension; i.e., an ideal body can be ascribed a four-dimensional shape.

The generic notion of *ideal objects* will for a while simply be taken for granted, but in later sections the first part, *ideal*, will be explicated. If I were not allowed to insert this term in the necessity-claims I will put forward, I guess they would look extremely odd. But the second part, *object*, is a primitive and undefined notion in the wide sense that Bunge speaks of objects (Bunge 1974, p.26). Both property bearers and properties are objects in this sense, and I will say no more about it.

The statement $A_{CM}1$ below I regard as a first axiom (A) in what I would like to call a *mechanism geometry* for cogwheel mechanisms (CM). In none of the axioms and theorems that follow are any operating forces mentioned; otherwise my term 'geometry' would be wide off the mark. Here is the first necessity-claim:

> ($A_{CM}1$) Necessarily, two directly connected ideal circular cogwheels do when rotating rotate in opposite directions.

This implies that if the rotational forces in a real corresponding mechanism with very rigid cogwheels are such that the wheels are made to rotate in the *same* direction, then this can nonetheless not happen. Instead, something, whatever it is, must happen to the force sources or what links these to the cogwheels.

If we change the mechanism by making one of the circular wheels larger than the other, then we can state another axiom:

> ($A_{CM}2$) Necessarily, if two directly connected ideal circular cogwheels are not equally large, then the one with the shortest diameter must when rotating rotate faster than the other.

Extending the first mechanism to three circular cogwheels we can derive the two theorems (T) below. I regard them as being analogous to, for example, Euclid's theorem that the three interior angles of a triangle are equal to two right angles (Euclid 2017, p.24; proposition 32). Here they are:

($T_{CM}1$) Necessarily, in a linear chain of three connected ideal cogwheels, the third rotates in the same direction as the first.[6]

($T_{CM}2$) Necessarily, three mutually and directly connected ideal cogwheels cannot rotate.[7]

Extending the mechanisms to non-circular cogwheels we can find quite a number of non-possibility axioms of the following kind:

($A_{CM}n$) Necessarily, a circular ideal cogwheel cannot form a rotational mechanism together with an ideal "cog-square."

For illustrations of non-circular cogwheels that *can* constitute a mechanism, see Fig. 2.

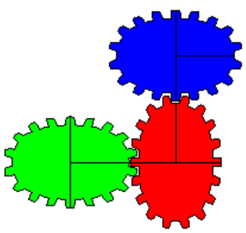

Fig. 2

Just like Euclid's axioms and theorems, the ones that I have presented do not contain numerical variables for real numbers. This is not an accidental manoeuver of mine. Since the axioms are concerned with rotating *shapes*, and this property dimension has not yet been quantified the way property dimensions such as length, mass, and energy have, they cannot at the moment be given the form of mathematical equations.[8] This fact is not in conflict with Bunge's views about mechanisms; according to him, mechanisms can be modeled without equations (Bunge 1997, p.423).

My simple examples of cogwheel mechanisms can easily be complemented by much more complex mechanisms with ideal bodies of both circular and non-circular shapes. Mechanical clocks being the first examples that come to my mind (Fig. 3). However, for the purposes of this paper, I need not discuss complex mechanisms. My present aim is only to defend this minimal thesis: *certain mechanism models contain necessary truths*.

[6] According to $A_{CM}1$, if $W_1$ rotates clockwise, $W_2$ rotates anti-clockwise, and if $W_2$ rotates anti-clockwise, $W_3$ rotates clockwise just as $W_1$ does.

[7] According to $A_{CM}1$, if all the three wheels are mutually connected and rotating, then one of the wheels must be able simultaneously to rotate in two opposite directions, which is a contradiction.

[8] In fact, I have the much stronger view that the property dimension of shape cannot be quantified. See (Johansson 2011), where a proof to this effect is given. So far, no one has been able to find a mistake in the proof, which relies on transfinite mathematics.

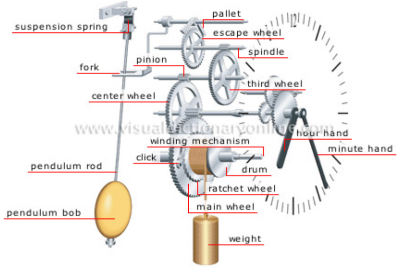


Fig. 3

I will, however, in the next two sections show that ideal bodies are not the only kind of ideal objects that can figure in mechanism models.

**Archimedes' Water Screw Mechanism**

Let us take a look at Archimedes' famous water screw (Fig. 4). Normally, it is presented as a device for moving water, and it consists of ordinary rigid bodies that do not let water through. When the screw is made to rotate clockwise, the water moves from the lower basin to the higher. But what is here true of water is true of most liquids, so in what follows I will call it Archimedes' liquid screw mechanism.

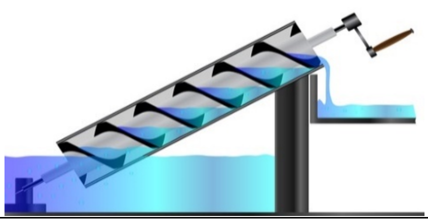

Fig. 4

The same kind of thought variations of the material that I made in relation to the cogwheel mechanism, can also be made in relation to Archimedes' screw mechanism. I divide the mechanism into three parts: the cylinder, the screw, and the liquid. When within the screw-in-the-cylinder, the liquid takes on the shape of a spiral formed pillar.

First case, the cylinder and the screw are made of a weak and porous material that lets many liquids through, and the liquid is water. In this case, not all of the water will be moved from the lower to the higher place; partly because the screw may break, and partly because some of the water will move through the cylinder into the outside of the mechanism. That is, in this case the mechanism doesn't function properly even though part of the liquid will be translocated.

Second case, the cylinder and the screw are made of steel, but the liquid has such a high viscosity (= resistance to shape deformation) that it is incapable of taking on the spiral shape it has to have in order to fill the volume between the screw and the cylinder. That is, neither in this case does the mechanism function properly, even though part of the liquid will be translocated.

Third case, the cylinder and the screw are again made of steel, but the liquid has now such an extremely low viscosity that some of it falls apart into droplets when it is to enter the screw. Again, the mechanism doesn't function properly, even though part of the liquid will be translocated.

Fourthly, let us see whether we can find objects that are analogous to the ideal cogwheels, i.e., objects which make Archimedes' mechanism function properly by necessity.

The first change to make is quite obvious: let us regard both the cylinder and the screw as being absolutely rigid (ideal) bodies without holes. But, in fact, a similar change can be introduced in relation to the liquid-volume that is meant to be moved through the cylinder. I will alternately call this idealization an *absolutely malleable liquid-volume* and an *ideal liquid-volume*.

Liquid-volumes are often regarded as being the same and as having the same volume when they are poured from one kind of container into another; for instance, from a pot to a bottle, or from a bottle to a glass. The liquid-volume is regarded the same in spite of the radical shape changes it undergoes in the process. By stipulation, I now define an absolutely malleable (ideal) liquid-volume as a liquid-volume that:

(i) can take on any shape
(ii) cannot possibly change volume (size)
(iii) cannot possibly lose parts or take up new parts
(iv) cannot possibly be destroyed.

The defining characteristic (iv) secures that the individual liquid-volume in question has an enduring individual (numerical) identity during the processes it partakes in. If we regard liquids as aggregates of molecules, then neither statement (iii) nor (iv) can possibly be true. That is beside the point though, since it is an ideal object that is defined.

Characteristic (i) introduces a feature that cannot be accepted if ideal objects are regarded as Platonic objects; by definition, Platonic objects are atemporal, and so by definition non-changeable. However, when ideal objects are regarded as fictions in the way I will propose and defend in later sections, then even temporal ideal objects become acceptable.

If we conceive of Archimedes' liquid screw mechanism as consisting of absolutely rigid (ideal) bodies that move an absolutely malleable (ideal) liquid-volume, we have in thought constructed what adequately can be called Archimedes' *ideal* liquid screw mechanism (AM). About this we can truly state this axiom (A):

($A_{AM}$1) Necessarily, when the ideal screw is rotating in the ideal cylinder, the ideal liquid-volume moves from one of the possible positions to the other.

Note that in the axiom there is no mention of a lower and a higher place between which the liquid is moved. The reason is that such talk would bring in associations to gravitational forces that make the liquid of itself move downwards. But all the objects involved are assumed to be as massless, colorless, and outside of all causal factors as the figures and solids of Euclidean geometry are.

## The Piston Mechanism

The steam engine was central to the first industrial revolution, and the internal combustion engine to the second. Common to both kinds of engine is that they contain – and essentially so – a piston mechanism, see Fig. 5. The very piston itself is contained in a cylinder in which it can move back and forth. There is a gas in the cylinder, and when it expands (in the internal combustion engine because the gas is ignited and explodes) the piston moves. It is only this movement that, in the light of the preceding sections, I will say some brief words about.

Fig.5

Of course, whether a real piston mechanism will function or not depends on the materials of the cylinder and the piston, and on what the gas is like. This time, in contradistinction to the cogwheel and the liquid screw mechanisms, I will immediately ask whether in relation to the mechanism it is possible to introduce ideal objects that give rise to a necessary truth.

To begin with, the cylinder and the piston can be regarded as absolutely rigid (ideal) bodies without holes, and I postulate that the piston without friction can freely move inside the cylinder. But what about the gas? Let me introduce still another idealization. By stipulation, I define an *absolutely gaseous (ideal) gas-substance* as an object that:

(i) can take on any shape
(ii) can take on any volume (size)
(iii) cannot possibly lose parts or take up new parts
(iv) cannot possibly be destroyed.

As in the ideal liquid-volume case, characteristic (iv) secures that an individual ideal gas-substance has an enduring identity during the processes in which it partakes. And, again as with liquid-volumes, if we regard gases as aggregates of molecules, then the statements (iii) and (iv) cannot be true. However, the gas-substance defined is an ideal object whose parts are left outside the definition. This means that ideal gas-substances cannot be identified with aggregates of particles (e.g. molecules), which, in turn, means that the notion of an *ideal gas-substance* must not be conflated with the notion of an *ideal gas* (such a gas is constituted by perfectly elastic collisions between particles).

When an ideal gas-substance expands in the ideal cylinder-with-piston mentioned, then, necessarily, the piston moves in the cylinder. The process can be give the form of a piston mechanism (PM) axiom (A):

($A_{PM}$1) Necessarily, when the ideal gas-substance expands in the ideal cylinder, the ideal piston moves as long the gas-substance expands.

From an epistemological point of view, the necessary truths $A_{CM}1$, $A_{AM}1$, and $A_{PM}1$ are on a par; and they are trivially true. In the first case there are only two ideal bodies, in the second there are two ideal bodies plus an ideal liquid-volume, and in the third there are two ideal bodies plus an ideal gas-substance. Since neither ideal liquid-volumes nor ideal gas-substances have an enduring or changing shape of their own, they must be combined with ideal bodies in order to be able to have a specific function within a specific mechanism. By combining absolutely rigid bodies, absolutely malleable liquid-volumes, and absolutely gaseous gas-substances, one can create models of many ideal mechanisms; models that, like the ones presented, can contain necessary truths.

The definitions of ideal bodies, liquid-volumes, and gas-substances given do not allow them to be transformed into one another. But, of course, transformations of real bodies, liquids, and gases into each other are central to many real mechanisms. The purpose of this paper, however, I remind the reader, is not to discuss and understand mechanisms in general, only to defend the minimal thesis that certain mechanism-models contain necessary truths.

Now time has come to explicate the generic notion of ideal objects that I am using.

## To Reality through Fiction

The heading above is stolen from a heading by Bunge (2006, ch.8), and the reason behind my "theft" will become clear some paragraphs below.

Mostly, Bunge says (as already quoted): "Mechanisms are processes in concrete (material) systems." In his *Definition 1.5* (as also already quoted), on the other hand, he is more general and says only: "A *mechanism* is a set of processes in a system." This definition allows one to speak not only of concrete mechanisms, but also of *ideal mechanisms*. In the spatiotemporal world, I take it for granted, there are no absolutely rigid (non-subatomic) bodies, absolutely malleable liquid-volumes, or absolutely gaseous gas-substances. This is the reason why the necessary truths I have presented are about ideal objects. In what way then does the naturalist Bunge conceive of ideal objects?

Throughout the history of philosophy, pure mathematical numbers have been reckoned prototypical ideal objects, and as such also reckoned prototypical Platonic objects; that is, being regarded as existing non-spatially and non-temporally in a non-changing realm distinct from our common spatiotemporal universe and from our minds. By definition, no naturalist can accept Platonic objects, so what is Bunge's view of pure numbers? He claims they are fictions of a certain kind. I quote:

> Still, I submit that fictionism, while utterly false regarding factual science, is quite true concerning pure mathematics. [...] Consequently, the concept of existence occurring in mathematical existence theorems is radically different from that of real or material existence. [...] In short, mathematicians, like abstract painters, writers of fantastic literature, "abstract" (or rather uniconic) painters, and creators of animated cartoons deal in fictions. To put it into blasphemous terms: ontologically, Donald Duck is the equal of the most sophisticated nonlinear differential equations, for both exist exclusively in some minds. (Bunge 2006, p.192)

My own views on pure numbers are similar, even though they differ somewhat. These differences, however, are of no importance for this paper, and will not be discussed.[9] I will at the moment rest content with using Bunge's views on numbers as a foil for my discussion of ideal mechanisms. One of my points about the latter is analogous to Bunge's about numbers. I claim that *we can go to real mechanisms through fictional mechanisms*.[10] Or, in other words, we can sometimes obtain a kind of knowledge of features of real spatiotemporal mechanisms by means of necessary truths about ideal fictional mechanisms. How I think this works will be explained in a later section; first more words about numbers.

In the world we perceive, it is impossible to point at pure numbers such as 1, 2, 3, 4, 5, etc. We can only point at their symbols, i.e., the corresponding numerals. I have just used the first five numerals of the decimal system (which has ten as its base); if I had used the binary system (having the base two) I should have written "pure numbers such as 1, 10, 11, 100, 101, etc." One and the same number may in number systems with different bases be represented by different numerals. Before mathematics was invented, there were not even numerals to point at. Conclusion: numbers are not mind-independent spatiotemporal objects.

For thoroughgoing naturalists, like Bunge and myself, who find both nominalism, psychologism, and many-worlds thinking incoherent, talk of pure numbers must be regarded either as complete nonsense or as being about fictional objects of some kind; that is, be about man-made objects without being reducible to mental states. But to regard mathematics as nonsense is of course out of the question. Therefore, the only option left is to regard pure mathematical numbers as being fictional objects of some sort. Alternately, Bunge also calls *fictional* objects *abstract* or *conceptual* objects: "mathematics […] deals exclusively with ideal (or abstract or conceptual) objects" (Bunge 2006, p.28).

In conformity with Bunge's views, I will now introduce two distinctions. First, Bunge's general bipartite distinction between *real objects* and *conceptual objects* (Bunge 2006, p.27). Real objects exist in real space and time, but purely conceptual objects exist only in fictional discourses. The latter objects have as such no *real* spatiotemporal existence, even though the fictional discourse in which they exist may allow a fictional spatiotemporal framework; normally, fictional novels tell a narrative that unfolds in both space and time.

Second, I will divide conceptual objects into two sub-kinds, ideal and non-ideal (e.g. literary) fictions. According to Bunge, real, ideal, and literary objects can when referred to in propositions figure in three corresponding kinds of truths: *factual*, *formal*, and *artistic* truths, respectively (Bunge 2006, p.194).

Bunge claims that "Logic, philosophical semantics, and mathematics are formal sciences," and that as such they can contain formal truths (Bunge 2006, p.193). If I would use this terminology, then I would have to say that the mechanism axioms I have presented are formal truths. However, in an obvious sense they have contents. Therefore, I have chosen to call them necessary truths in spite of the fact that they must be reckoned formal truths in Bunge's conceptual apparatus.

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[9] My views are put forward in (Johansson 2013) and (Johansson 2015). The way I differ from Bunge is presented in sect.2.2 of the first paper.

[10] There have in this decade been some discussions about whether certain scientific models should be regarded as fictions or as analogous to fictions. See for instance Contessa (2010), Frigg (2010), Toon (2010), Morrison (2015), and Bueno et al. (2018). But in none of these papers and books is my thesis that certain models can contain necessary truths put forward.

The distinction between ideal and non-ideal fictions is of utmost importance in order to understand Bunge's (and my) conception of the ontological status of numbers. Of course, if both pure numbers and fictional figures in novels and cartoons are fictions, then there must be some radical and important difference between them. As repeatedly pointed out in the philosophical discussions of fictions in literature and cartoons, conceptual and pictorial objects like Donald Duck are *ontologically indeterminate*. That is, as long as the creators of Donald Duck do not, for example, tell exactly how old he is, what his weight is, etc. – and that by so-called conversational implicatures such descriptions do not follow from what has been explicitly said – there simply are no corresponding facts of the matter. The natural numbers, on the other hand, cannot possibly have such ontological spots of indeterminacy.[11] No mathematician can freely add a feature to the posited natural numbers the way the creators of Donald Duck can freely fill in what hitherto has been a spot of indeterminacy.

This difference between ideal and non-ideal fictions does by no means imply that number theory cannot contain unsolvable problems. Trivially, where there are spots of ontological indeterminacy there are unsolvable epistemological problems; but the converse is not true.

Bunge says as follows: "Indeed, it [his kind of fictionism] distinguishes between mathematical fictions on the one hand and myths, fairy tales, theological speculations, abstract paintings, parapsychological and psychoanalytic phantasies, as well as many-worlds philosophical theories, on the other" (Bunge 2006, p.193). After having asked "how does the fundamental theorem of algebra differ from the claim that Superman can fly or Mickey Mouse can speak?," he lists twelve points in which he regards the difference to manifest itself (Bunge 2006, p.204), but for the purpose of this paper they need not be discussed.

Briefly put, Bunge's view that we can go "to reality through [mathematical] fiction" can be reached by an inference of the following kind:

(P1) Modern physics gives us knowledge of mind-independent reality
(P2) In much of modern physics, mathematics is an indispensable part of the content
(P3) Pure numbers are fictions
Hence:
(C) We can go to reality through fictions.

I have no qualms about the inference. However, the view is in need of a credible description of what such pure-numbers-to-reality transitions are like. One problem can be stated thus: natural laws are *not* necessary truths, but mustn't applications of mathematical necessary truths give rise to new necessary truths? Bunge does not say much about the transition, but I will outline an answer that I find both reasonable and as not being contradicted by anything I know Bunge has claimed.

Look at the following simple true arithmetic statement: 'necessarily, 1 + 1 + 1 = 3'. Since the numbers are not to be found in spatiotemporal reality, there must be some intermediary level that is at work before we can state, for example: 'necessarily, 1 tomato + 1 tomato + 1 tomato = 3 tomatoes'.

In order to get a concrete case, let us think of a child called Adam, who has just learned to count and add a little. His parents want to profit from his new abilities, and they ask him to go to the nearby shop and buy three apples. The following scenario then unfolds.

[11] The expression 'spots of indeterminacy', which I like very much, is taken from Roman Ingarden; see (Johansson 2010, p.94) and (Johansson 2013, pp.29-30).

In the shop, Adam goes to a shelf full of tomatoes and takes first one tomato and puts it in his bag, then a second, and so at last a third. When doing this he says to himself: “one tomato, plus one tomato, plus one tomato; now I have three tomatoes.” If we symbolize this real-world adding by the symbol ‘$+^r$’ and the real-world result by ‘$=^r$’, then we can represent Adam’s procedure by the expression ‘1 tomato $+^r$ 1 tomato $+^r$ 1 tomato $=^r$ 3 tomatoes’. This *physical addition* is in contradistinction to the corresponding arithmetic addition not a necessary truth, which will soon become evident.[12] When Adam comes home, and proudly wants to show that he has been able to do what was expected of him, it turns out that on his way home his arms have been too movable; the bag has bumped into various walls and posts. In the bag there is now only a certain amount of tomato porridge; no distinct tomatoes can be discerned.

In the preceding sections I have introduced non-mathematical idealizations, and such a move can be made even here. By stipulation, I now define an *ideal tomato* as a tomato that:

(i) can take on some (but not all possible) different shapes
(ii) can take on some (but not all possible) different sizes
(iii) can lose some parts and take up some new parts
(iv) cannot possibly be destroyed (and so lose its individual identity).

So defined, the fourth characteristic of an ideal tomato makes it, like Plato’s atemporal tomato-idea, an indestructible tomato, but unlike Plato’s idea it exists in a temporal framework. Nonetheless, as can be seen from the corresponding definitions, ideal tomatoes differ radically from absolutely rigid bodies; the former can change, the latter cannot. If Adam had been able to pick ideal tomatoes and put them into his bag, then, surely, he would have brought home three distinct tomatoes independently of what his bag would have bumped against during the walk.

As far as I can see, the best way to understand the transition from the purely arithmetic expression ‘1 + 1 + 1 = 3’, which represents a necessary truth, to the expression ‘1 tomato $+^r$ 1 tomato $+^r$ 1 tomato $=^r$ 3 tomatoes’, which at best represents a contingent truth, is to introduce *additions of ideal objects*. Instead of distinguishing only between, on the one hand, pure arithmetic numbers and arithmetic addition, and on the other real-world tomatoes and physical addition, we should posit intermediate strata and additions of objects within these. I think there are four kinds of proposition strata and three kinds of between-strata relations to be taken into account:

(d) Necessarily, 1 + 1 + 1 = 3
(c) Necessarily, 1 indestructible object + 1 indestructible object + 1 indestructible object = 3 indestructible objects
(b) Necessarily, 1 ideal tomato + 1 ideal tomato + 1 ideal tomato = 3 ideal tomatoes
(a) Contingently, 1 tomato $+^r$ 1 tomato $+^r$ 1 tomato $=^r$ 3 tomatoes

Top-down, the relation between the strata (d) and (c) is a straightforward case of *application*; if you apply a necessary truth, the result should also be, as here, a necessary truth. Bottom-up, the relation can be seen as a kind of *abstraction*; the indestructible objects are abstracted away in the sense of being disregarded. There are interesting things to discuss about

[12] The notions of *physical addition* and *physical sum* can be found in (Bunge 1967, pp.199-202). I have earlier discussed physical and pre-mathematical addition in (Johansson 1996).

both this top-down and this bottom-up relation, but I will not delve into these.[13] The relation between stratum (b) and stratum (c) is that of logical *subsumption*; an ideal tomato is simply by the definitional characteristic (iv) made into a kind of indestructible object.

The most interesting question for this paper is this: How can there be a relation between the necessary truth (b) and the factual truth (a)? My answer is: because real objects can be *approximations* of ideal objects. The term 'approximation' is meant in a wide and informal sense. Ideal tomatoes are absolutely indestructible, but all real tomatoes can be destroyed. In this sense, there is an ontological gap between them. Nonetheless, different real tomatoes can be ranked as in a certain situation being more or less easily destroyable, and also as in and of themselves being more or less prone to decay. Therefore, despite the ontological gap, certain real tomatoes can be more similar to the ideal tomato than some others are, i.e., be better approximations of the ideal tomato than the others.

The view I have presented must not be conflated with the view that we regard some real tomatoes *as if* they *are* ideal tomatoes. When I count real tomatoes, I do not look upon them as if they are indestructible; I look upon them as for some time probably enduring, i.e., for some time probably approximating ideal tomatoes.

My explanation of what it looks like to go from purely arithmetic (ideal and fictional) addition to physical addition can easily be extended from the noun 'tomato' to all so-called count nouns. Now, whether this explanation is right or wrong, I will take it for granted when later I will look at the relationship between ideal and real *mechanisms*. In the next section, I will in passing say some words about approximations between factual theories that contingently are true or false. They cast some further light on the informal approximation notion I rely on.

**Interlude: Bunge and Popper on Approximations to Factual Truths**

In the twentieth century, it was quite common among physics teachers to say things such as these: (i) from the perspective of special relativity, Newtonian mechanics still gives rise to *approximately true* predictions for bodies whose velocity is small compared to the velocity of light, and (ii) from the perspective of Newtonian mechanics, Galilei's law for falling bodies still gives rise to *approximately true* predictions for bodies falling near to the earth. In spite of this, mainstream analytic-philosophical philosophy of science was very skeptical to notions such as 'approximate truths', 'partial truths', 'degrees of truth', and 'truthlikeness'. Bunge and Popper are the two outstanding great dissidents from this orthodoxy. If I may call them so. They think, and so do I, that a notion of approximation between factual propositions/theories is not only meaningful, it is also needed in order to understand and further the development of science. I quote Bunge:

> Contrary to a widespread opinion, scientific realism does not claim that our knowledge of the outer world is accurate: it suffices that such knowledge be *partially true*, and that some of the falsities in our knowledge can eventually be spotted and corrected. (Bunge 2006, p.30; italics added)

That a factual statement is partially true means, that when it is compared to a factual statement that is (for the moment) regarded as literally true, it (i) is literally false, but (ii) has nonetheless quite a similarity to the true statement.

[13] I discuss them in (Johansson 20l5, sects.6-7), and I think they point towards what might be called a property view of the natural numbers.

Looking back at the philosophy of science since the 1960s, I have the impression that Bunge's conception of approximate truth did not give rise to such a widespread discussion that Karl Popper's for some decades did. My explanation is this.

In Bunge's book *Scientific Research*, the general notion of partial truth is rather informal: "It is only some logicians who still oppose the very idea of partial truth, as a result of which we continue using an intuitive, presystematic concept of partial truth" (Bunge 1967, p.301). Later, in his *Treatise of Basic Philosophy*, he gives an implicit definition of the formal features of 'degrees of truth' by means of a number of postulates (Bunge 1974b, ch.8.3.2-3).

Popper, on the other hand, did for a long time (probably from the end of the 1950s to the end of the 1970s) wrongly believe that much more could be done. During this period, he claimed that the notion of approximate truth could be given an explicit definition and well-defined measure, too. This claim gave rise to wide discussions. Most philosophers of science at the time thought that without a precise explication the expression 'approximate truth' would be meaningless. When, at last, consensus was reached that such a definition or measure is impossible to construct, most of those involved in the discussion dropped the whole notion. But not Popper. He still defended the importance of retaining the original informal notion. For this history, and a further elaboration and defense of the informal notion of approximate or partial factual truths, see my (Johansson 2017).

In the earlier sections, I have without any arguments taken an informal notion of approximation for granted, but the arguments needed can be found in the paper just mentioned. In the sections below, I will continue to use the same informal notion. I think the notions 'approximate truths' and 'objects approximating each other' imply each other. If there are statements that are approximately true, they must contain some concepts whose referents are only approximations of these concepts; and if there are concepts whose real-object referents are only approximations of certain concepts, then statements containing these concepts can be created.

Let me add that I regard the notion of approximate truth as a *complement* to the simple binary opposition between being true and being false. The latter opposition is needed in order to understand what a valid deductive inference is, the former notion is needed in order to understand what an empirical-scientific development is. I end this section with another quotation from Bunge:

> And yet, paradoxically, no one seems to have produced a detailed, true, and generally accepted theory of objective and partial truth. We only have a few insights into the nature of factual truth, the way its knowledge emerges from tests, and the confluence of truths attained in different fields. (Bunge 2003, p.249)

**To Real Mechanisms through Ideal**

Models and descriptions of real mechanisms can only be contingently true, but models of ideal mechanisms can, as I hope to have shown, contain necessary truths. How, then, can the latter kinds of models shed light on real mechanisms? My answer aligns of course with my views on how arithmetic truths can be applied to ordinary kinds of objects. Bunge says:

> models are seldom if ever completely accurate, if only because they invite more or less brutal simplifications, such as pretending that a metallic surface is smooth, a crystal has no impurities [etc.]. These are all fictions. However, they are all stylizations rather than wild fantasies. Hence,

> introducing and using them to account for real existents *does not commit us to fictionism*. (Bunge 2006, pp.189-190; italics added)

Here, unusually, I think Bunge moves to fast. Whereas he claims that fictionism is true for mathematics, he can in the quotation easily be interpreted as denying that fictionism can ever be true for models. In my opinion, what should be said is this:

> Fictionism is as true for *ideal* mechanisms as it is for arithmetic addition, and it is as false for *real* mechanisms as it is for physical addition.

That Bunge himself doesn't say so is, I guess, due to the fact that he never has thought about the possibility of fictional but nonetheless ideal mechanisms.

Before proceeding, let's go back to Plato. Even he can be said to have worked with a relation of approximation. He distinguished between a realm of ideas and the spatiotemporal sensible world in which we normally live. The spatiotemporal objects, he claimed, do *participate more or less* in corresponding objects in the world of ideas. When this assumption is adhered to, it becomes natural to say that spatiotemporal objects are better or worse approximations of the posited ideas or abstract objects. Euclidean geometry affords a good example. In the idea of the circle the circumference is exactly the diameter multiplied by π, but in all spatiotemporal circles this is only approximately the case. Similarly, in the idea of a triangle the sum of the interior angles is exactly 180 degrees, but in all spatiotemporal triangles this is only approximately so. Actual macro- and mesoscopic spatiotemporal circles and triangles can only more or less approximate the ideal figures in the realm of ideas (I leave it for physics to decide about the shapes of microscopic particles).

On the approach I am defending, there is no problem in handling real everyday spatiotemporal circles and triangles the way I have analyzed additions of spatiotemporal tomatoes. Now, however, the strata (c) and (d) are not needed. The schema below is enough:

(b) Necessarily, in *ideal* triangles, first angle + second angle + third angle = 180 degrees
(a) Contingently, in *real* triangles, first angle $+^r$ second angle $+^r$ third angle $=^r$ 180 degrees

With this in mind, let us look at mechanisms. Even though there are no absolutely rigid bodies, absolutely malleable liquid-volumes or absolutely gaseous gas-substances in the spatiotemporal world, there are rigid bodies, malleable liquids, and gaseous substances. Moreover, such rigidity, malleability, and gasousness can take degrees. Therefore, these properties can more or less approximate the corresponding absolute property. For instance, the steel wheel in the cogwheel example is closer to absolute rigidity than the wool wheel and the glass wheel are. In the mechanism examples presented, there is no mathematics, so there is no analogy to be made with the four-strata arithmetic tomato example, but the two-strata Euclidean triangle example affords the analogy searched for:

(b) Necessarily, two directly connected *ideal* circular cogwheels do when rotating rotate in opposite directions.
(a) Contingently, two directly connected *real* circular cogwheels do when rotating rotate in opposite directions.

Since real cogwheels can approximate ideal cogwheels, there is, when approximation obtains, a relation between the strata (a) and (b). Let it be noted, that since the ideal wheel is a man-made fiction, the approximation relation is here a relation between something real and something fictional. The relation is an internal relation in the sense that *if* both the relata exist, then, necessarily, the approximation relation obtains; it then comes so to speak for free. Compare the relation between these two spots: ● •. *Given* the existence of both, necessarily, the spot to the left is larger than the one to the right, i.e., the relation *larger than* is an internal relation in the sense meant.

Of course, as there can be approximation relations between *fictional ideal* and real mechanisms, there can also be such relations between *fictional non-ideal* mechanisms and real mechanisms. An inventor can first in thought invent either an ideal mechanism or a non-ideal mechanism, and then later try to create in the world an approximately similar mechanism. I have in this paper chosen to stress the existence of fictional ideal mechanisms, but in technology, probably, fictional non-ideal mechanisms are of more importance.

What has just been said about mechanisms in relation to inventors, is also applicable to researchers. A researcher can first in thought conjecture either an ideal mechanism or a non-ideal mechanism, and then later try to test whether the world contains an approximately similar real mechanism. This is true in relation to both non-living nature, living nature, and social reality.

I have made no attempt to relate my views on ideal mechanisms to the general discussion about idealizations in the philosophy of science, but I would like to make a brief remark in relation to Max Weber's famous notion of *ideal types* (or ideal-type models).

Weber was both a philosopher of science and one of the founding fathers of sociology, and he used the notion of ideal types in order to defend the view that sociology can be a science. In the light of my distinction between ideal and non-ideal fictional mechanisms, all his ideal types should probably be classified as non-ideal fictions, but it might also be argued that he simply never considered such a distinction. Be this as it may, in my opinion both the ideal and the non-ideal mechanisms I have spoken of can function the way Weber intended his ideal types to function. His view was that many social patterns and structures can only be grasped conceptually by being seen through the artificial lens of an ideal type.[14] I think the same is true of many real natural and technological mechanisms. In order to be able conceptually to communicate about many of them, we need ideal types.

From the Weberian perspective hinted at, ideal mechanisms are ideal types that can function the way mathematics and classic pre-analytic Euclidean geometry function when applied to real-world phenomena.

## Abstract Objects as Fictions

From Plato and ancient Neoplatonism, via a number of medieval Islamic and European philosophers, up until Frege and much of contemporary world-wide analytic philosophy, abstract objects are regarded as having a necessary existence. Hereby, they are also ascribed a mind-independent existence. To me, it is a bit of a mystery how contemporary philosophers can have such a view of the abstract objects they work with, be it numbers, sets, propositions, or all of these, and at the same time call themselves naturalists, which many of them do. Normally, being

[14] For a comprehensive discussion of Weber's ideal types, see (von Schelting 1934). He finds distinct kinds of ideal types in Weber. Along one dimension, von Schelting divides ideal types into "causal-real" and "non-causal ideal," and along another into "generalizing" and "individualizing."

a naturalist means thinking that there is a mind-independent reality with spatiotemporal objects, but that there is no reality such as a God, Platonic objects, or Kantian transcendental faculties. For a naturalist, abstract objects cannot have a necessary existence; they must either be regarded as being directly immanent in the mind-independent spatiotemporal world, or as being man-made (or perhaps animal-made) constructions. And I think that our constructions of abstract objects have such a character that they deserve to be called fictions, even though this semantic option implies a widening of the traditional notion.

As I have already said, a distinction has to be made between ideal fictions and the non-ideal fictions in novels; numbers, sets, and propositions do not belong to the extension of the traditional notion of fiction. However, I will only stress this fact, not say more about it, even though there are many details to be discussed.

Traditionally, the term 'fiction' is connected both to what we experience in dreams and to what we are presented to in novels. When we stress dreams, all fictions easily look purely personal, but the fictions in novels are normally regarded as intersubjective social constructions. From the point of view of mind-independent nature, as well as from the point of view of transcendent realms, all fictions, both personal and social, are in a straightforward sense just fictions. However, from the point of view of social reality, social fictions are simultaneously also real. That is, they belong for some time to some social reality. The expression 'socially real fictions' is not a contradiction in terms.

As a matter of fact, we can in many conversations and writings identify and re-identify fictional objects of various sorts; both temporally across the mental states of ourselves and inter-subjectively across the mental states of ourselves and others. This should be taken as a fact that philosophical investigations should elucidate and explain, not as something that philosophy can show to be an illusion. Therefore, in this paper, I have taken the existence of fictions as social facts for granted.[15]

Since a social fiction is in some sense individually the same in all the mental states in which it exists, it cannot be reducible to a class of mental states, even though it is the case that it would not exist at all without mental states directed at it. Should they then be called subjective or objective objects? About the ideal fictions of mathematics, Bunge says:

> the statements in pure mathematics are not ontologically objective: they do not refer to the real world. But of course they are not subjective either: they do not report on the speaker's state of mind. Thus, they are neither objective nor subjective in an ontological sense, even though they are impersonal and asocial [in the sense of being socially neutral]. (Bunge 2006, p.190)

I have earlier quoted Bunge saying that "paradoxically, no one seems to have produced a detailed, true, and generally accepted theory of objective and partial truth." In analogy with this, I would now like to say: paradoxically, no one seems to have produced a detailed, true, and generally accepted theory of what is neither objective nor subjective in an ontological sense.[16]

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[15] My analysis of how it can be possible for different persons to refer to the same individual fiction is presented in (Johansson 2010). It differs from Bunge's, but it also differs from "the pretence theory of fiction" and "the make-believe theory of fiction." Both the latter have been used in discussions of fictional models in science; see Frigg (2010) for the first and Toon (2010) for the second.

[16] In the sixth section, I briefly compares Bunge's and Popper's conceptions of approximate truths. A comparison between Bunge's view of what is "neither objective nor subjective in an ontological sense" and Popper's view of his so-called "world 3" would lay bare a further striking similarity between them.

This notwithstanding the fact that, in the last decades, social ontology has become more or less a philosophical discipline of its own. There, however, ideal objects are never discussed.

Since I believe in the existence of certain kinds of non-empty necessary truths, and since I am a Bunge-like anti-Platonist and anti-Kantian naturalist, I will end by putting forward a question that despite my naturalism has a Kantian ring. Kant took the existence of synthetic a priori-truths for granted, and asked how they are possible. In my opinion, we must nowadays take it for granted that abstract objects can be given a place in a naturalist framework. Therefore this question: *how are abstract objects, e.g. numbers and ideal mechanisms, possible?*

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Figure 1

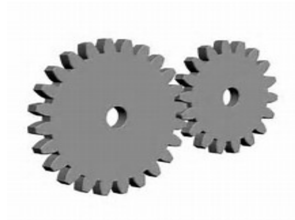

Figure 2

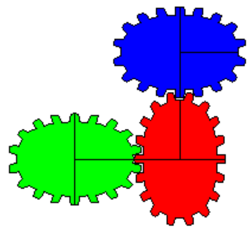

Figure 3

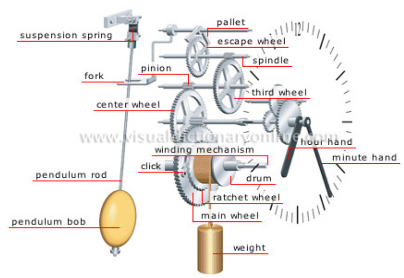


Figure 4

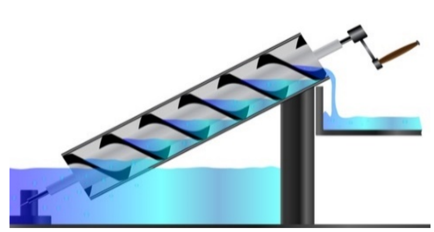

Figure 5