Scattered Exemplification and Many-Place Copulas

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Abstract Can there be relational universals? If so, how can they be exemplified? A monadic universal is by definition capable of having a scattered spatiotemporal localization of its different exemplifications, but the problem of relational universals is that one single exemplification seems to have to be scattered in the many places where the relata are. The paper argues that it is possible to bite this bullet, and to accept a hitherto un-discussed kind of exemplification relation called 'scattered exemplification'. It has no immediate symbolic counterpart in any Indo-European natural language or in any so far constructed logical language. In order to remedy this, a notion called 'many-place copula' is introduced, too.

Keywords Exemplification · Relations · Plural Predication · Copula

This paper tries to develop views about the existence of what can be called the 'scattered exemplification' or 'scattered instantiation' of relational universals. I have earlier put forward the idea in the paper "Hypo-Realism with Respect to Relations" (Johansson 2011b). The prefix 'hypo-' is intended to indicate that relations of the kind I am discussing are less fundamental than what is truly fundamental in our spatiotemporal reality, and therefore metaphorically 'below' the basic entities.

I have to start with a word about terminology. The relation between a universal and its spatiotemporal particularities is by realists sometimes called 'exemplification' (Grossmann 1983, 1992) and sometimes 'instantiation' (Armstrong 1978, 1997). Some realists make a

simultaneous use of both terms. E. J. Lowe talks about an instantiation relation between property universals and property instances (and between substantial universals and substantial particulars), but about an exemplification relation between property universals and things; between the particulars property instances (modes) and things he posits an inherence relation (Lowe 2006). In what follows, I will use the term 'exemplification' for all relations that unites a universal of some kind with some kind of spatiotemporal particularity.

The exemplification relation has always, both in transcendent and immanent realism, been regarded as a two-term relation that relates the unity of a universal to a non-scattered particular spatiotemporal unity. Predicates such as 'flock' and 'swarm' have tacitly been assumed to be reducible to predicates about natural kinds that have the unity-to-unity exemplification relation mentioned. Let me symbolize it $_{U}Ep$; meaning – in three equivalent statements – that a universal U is exemplified in the particular p, that p exemplifies U, and that p is an exemplification of U. If, however, this is the whole truth of exemplification, then it seems to me impossible to regard relations (R) such as *taller than*, *larger than*, *heavier than*, *warmer than*, and *brighter than* as universals. Since an exemplification of R must involve two particulars a and b, and a state of affairs aRb, the relatum p of the exemplification relation in $_{R}Ep$ must in some way or other be the scattered collection of a and b. Therefore, let me symbolize the whole state of affairs as $_{R}E(a,b)$ or $_{R}Ep_2$, where the subscript means that p_2 represents a collection of two particulars; in case of 3-term relations, it becomes $_{R}Ep_3$, and in case of n-term relations $_{R}Ep_n$. This fact is the main reason why many, perhaps most, realists deny that such relations can exist in things the same way monadic universals can (more about this in Section 2). Either the relations are completely denied a mind-independent existence (Heil 2005, 2009), or they are regarded as what Armstrong calls 'ontological free lunches', i.e., they are said to exist in the curious (to me contradictory) way that they exist without constituting an "addition of being (1997, p. 12)."

My view, to the contrary, is that there are mind-independent relations of the given sort, but that this fact is hard to understand, since they require a special kind of exemplification relation. On the universality-side, this kind of exemplification relation has a relational universal as the relatum, but on the particularity-side it has a number of scattered particulars as the relatum. This is what I call 'scattered exemplification', and have symbolized as $_{U}Ep_n$. The important thing to note is that E does not relate the universal U to each of the particulars p_1 to p_n separately, but to all of them as a plurality, i.e., the plurality of p_1 to p_n constitutes one single relatum for the exemplification relation.

Section 1 below contains some general comments on the relationship between ontology and language; Section 2 contains a summary of what I have earlier said about scattered exemplification of relations; and in Section 3 I relate my views to the on-going discussions about plural reference and plural non-distributive predication in natural languages and logic. In my opinion, sentences such as '*a* is taller than *b*' contain a plural reference and a plural non-distributive predication. In the whole paper, realism with respect to a number of monadic universals will be taken for granted.

1 Default Ontologization

Trivially, there is no other way to speak about the world than by means of symbols. In the early decades of the twentieth century, Bertrand Russell was in much of his philosophical work motivated by the belief that it is possible to create a logical language that mirrors the ontological structure of the world much better than the natural languages do. He sometimes called it simply a 'logically ideal language', but if logic is not to be identified with such an enterprise, it had better be called an 'ontologically ideal logical language'. Since the grammar and syntax of any language can be given different interpretations, I will introduce the notion of 'the default ontologization of a language' (or of part of it). In such an ontologization all terms that are not obviously only functional or syncategorematic (such as 'or' and 'if–then' in ordinary English) are interpreted as having a language-independent referent. I call it 'default' since qualifications that prohibit it can be added; it should not be regarded as belonging to the essence of any language. In terms of this notion, Russell can be described as attempting to create a logical language whose default ontologization better mirrors the structure of the world than the default ontologizations of the natural languages do.

Two symbols of first-order predicate logic seemed to Russell in this ontologization respect to be very good: the existential quantifier (∃) and the relation symbol *R* (in *aRb, Rab, Rabc*, etc.). Let me comment on the existential quantifier first.

Russell came with his theory of definite and indefinite descriptions to the conclusion that entities either exist or do not exist *simpliciter*; and his criticism of Alexius Meinong's view that there are different *modes of being* (existence, subsistence, and absistence) is often regarded as a fundamental event in the rise of analytic philosophy.[1] Now, since predicate logic has only one symbol for existence, the existential quantifier, he thought that in this respect predicate logic mirrors the world well. It is easy, however, to create Meinongian

existential quantifiers. One has only to add subscripts to the existential quantifier. Meinong could, for instance, have distinguished between the quantifiers $_{E}\exists$ and $_{S}\exists$, and stipulated that $_{E}\exists x\, Gx$ means that at least one *x* that is *G exists*, whereas $_{S}\exists x\, Gx$ means that at least one *x* that is *G subsists*, respectively; let it be added that Meinong himself only meant that states of affairs, not objects, can subsist (Findlay 1963, p. 73f). A default ontologization of ordinary first-order predicate logic says that everything that exists exist in the same way, but a default ontologization of a predicate logic with subscripts on the existential quantifier says that there are different modes of being, one for each subscript.

According to the Russell of logical atomism, the world consists of particulars with monadic properties (universals) between which non-reducible relations (universals) hold; see e.g., (Klement 2009). The symbol *Fa* of predicate logic symbolizes a monadic property and the particular in which it is exemplified, and the symbol *aRb* symbolizes two particulars and a non-reducible relation between them. The belief that everything in the world must have the structure *S is P* is according to Russell a philosophically wrong ontologization of the Indo-European languages that has to be resisted. It has, he claimed, misled Leibniz into conceiving his curious monadology (the world consists of a number of monads with properties, but between which there are no relations at all); and it led Spinoza, Hegel, and Bradley to their curious monisms (the world with its history is one big subject and substance in which all other phenomena inhere); see, in turn, (Russell 1974, pp. 575, 560, 703) and (Russell 1910). The relation logic that is part of first-order predicate logic should, he hoped, free all metaphysicians from such relation-less systems.

Russell's relation logic also entails a rejection of Aristotle's and the scholastics' basic category of the relative (*pros ti*), i.e., a rejection of the view that relations are properties (accidents) that, even though inhering in one substance only, are "toward another" property; see e.g., (Henninger 1989) and (Jansen 2006). According to Russell, the ideal (onto)logical form of the natural language sentence 'Simmias is taller than Socrates' is not the Aristotelian *S is P* ('Simmias is (taller than Socrates)'), but the *aRb* of predicate logic ('Simmias - taller than - Socrates').

Many social and political revolutions have had adverse effects which the original well-intentioned revolutionaries couldn't dream of could be caused. To my mind, it seems as if the revolution in Anglo-American philosophy that Frege and Russell inaugurated with the invention of first-order predicate logic, has had at least one repercussion they would strongly dislike. The widespread training of a couple of generations of philosophy students in first-order predicate logic, seems to have made many of them so accustomed to this logical

language, that they take it the way Russell accused Leibniz, Spinoza, Hegel, and Bradley to have taken the old subject-predicate logic, i.e., as something whose default ontologization gives us the true ontological structure of the world. Now, Frege believed to the end of his life in Platonic objective thoughts, and Russell in the existence of universals, but the default ontologization of first-order logic – taken in isolation from second-order logic – ends, I will show, in some kind of nominalism.

In first-order predicate logic, only the proper names such as *a* in *Fa* and the individual variables, *x*, *y*, *z*, etc., that can be bound by the quantifiers ∃ and ∀, seem to be referring expressions. The predicates, as the name says, seem in first-order logic only to be predicating, but if there are monadic universals, then some predicates must be regarded as simultaneously being both referring (to property universals) and predicating (of individuals). A reference to properties comes out explicitly in the syntactic structure of second-order logic, but is invisible in first-order logic. Moreover, the proper names and the individual variables of first-order logic have (since *a* is not necessarily *F* or any other property) as a default ontologization property-less particulars. This means that when used outside of purely formal disciplines they must refer either to so-called 'bare particulars' or to so-called 'trope particulars'. In this way nominalism becomes the default ontologization of first-order predicate logic; a fact further strengthened by the subscript-less existential quantifier. Nominalism is a one-mode-of-existence ontology.

Even though differing in some details, the point just made aligns very well with Barry Smith's paper "Against Fantology," which starts as follows:

> A dark force haunts much of what is most admirable in the philosophy of the last one hundred years. It consists, briefly put, in the doctrine to the effect that one can arrive at a correct ontology by paying attention to certain superficial (syntactic) features of first-order predicate logic as conceived by Frege and Russell. (Smith 2005, p. 153)

Another of the things Smith notes is, that since predicate logic contains no distinction between predication in the category of substance/kind ('John is a human being') and in the category of accident/property ('John is hungry'), it also easily makes ontologists (e.g., Armstrong) delete the ordinary language distinction between kinds of things and properties of things. Russell, by the way, explicitly deleted it, too. He wanted to get rid of the whole notion of 'substance' and its concomitant 'kind of substance'; see e.g., (Russell 1974, p. 211).

Smith also mentions the vanishing of the copula in predicate logic (ibid., p. 161), but he does not expand on its effects on ontology. This feature, however, is central to the topic of this paper. The default ontologization of the natural language sentence 'the table is round' brings with it not only the existence of a particular (denoted by 'the table') and the existence of a universal (denoted by 'round'), it also brings with it the existence of an exemplification relation (denoted by 'is'). If the sentence is formalized in Aristotelian term logic, it becomes *S is P*, and the three terms can be regarded as denoting three ontologically different kinds of entities. However, when it is formalized in first-order predicate logic, it becomes only *Fa*. Of course, whether 'the table' is symbolized *S* or *a*, and whether 'round' is symbolized *P* or *F*, is of no importance. What is important here is that in subject-predicate logic there is a copula, an 'is', that might denote a relation, whereas in predicate logic there is no corresponding term at all. In all probability, even this fact supports the tendency of many philosophers to ontologize first-order predicate logic into nominalism. Philosophers thinking only in terms of this logic cannot find a term that denotes a relation that connects the seeming or real generality of the predicate *F* with the particular denoted by the proper name *a*.

The last fact might also be a hidden reason why a philosopher such as Armstrong, who posits monadic property universals, is of the opinion that there is no relation of exemplification (instantiation). In his view, even though he uses the term 'instantiation', he claims that the relation between a universal and a particular is "closer than union (Armstrong 1978 II, p. 3)."

I have already pointed out above how easy it is to modify predicate logic into a logic that can speak of different modes of being. And it is just as easy to modify it in such a way that a symbol that might denote exemplification relations becomes part of it. All one has to do is to exchange the expression *Fa* for, say, ^{is}Fa.

I will now introduce a special symbolism that generalizes ^{is}Fa. Not in order to revive the search for an ontologically ideal language, but in order to make it easier to think about the topic of this paper: scattered exemplification. It is well known from mathematics that different symbolisms may make it easier or harder to think correctly about something. And I think that the default ontologizations of both contemporary natural languages and contemporary logical languages are such that even the logical possibility of scattered exemplification is hard to see. I will introduce a new kind of copula: *many-place copula*. There are, I will show, reasons to speak not only of the ordinary copula of Aristotelian term logic, but also of a two-place copula 'is—is', a three-place copula 'is—is—is', and so on. Two-place copulas give rise to grammatical and logical structures such as:

- Simmias ^{is}*taller-than*^{is} Socrates
- *a* $^{is}R^{is}$ *b* (= $^{is–is}$*Rab*)

If we do not simply hide or take away the copula as is done in the *Fa* of predicate logic, but write ^{is}*Fa*, we should write the relation logic formula *aRb* as *a* $^{is}R^{is}$ *b*, or as $^{is–is}$*Rab*; *R* is predicated of two logical subjects, *a* and *b*, not only one.[2]

When the notion of 'two-place copula' is generalized into that of 'many-place copula', then of course only the symbolism $^{is–is}$*Rab* can be used. A three-place predicate such as 'between' needs a three-place copula, and the sentence 'Plato is between Simmias and Socrates in length' can be written and symbolized as follows:

- $^{is–is–is}$*Between-in-length* Simmias, Plato, Socrates
- $^{is–is–is}$*Babc*

As I am using the term 'copula', it represents what connects a logical subject to a logical predicate. The distinction made between one-place copulas and many-place copulas must not be conflated with the linguistic distinction between the auxiliary verbs 'is' and 'are' of ordinary language. First, there is no way in which an 'are' can be meaningfully inserted in the two-copula sentence 'Simmias *is* taller than Socrates'. Second, many-place copulas can be connected to plural subject terms and take the linguistic form 'are', as in the sentence 'the basket players ^{are}*taller-than*^{are} the jockeys'. In the next section, I will present my views on scattered exemplification of what I call 'weakly internal relations', but in Section 3 I will return to the logical-linguistic issue just highlighted. I think it sheds some new light on the discussions about plural reference and plural non-distributive predication.

Without yet having made all the notions clear, I will nonetheless already here present a thesis put forward in Section 3: *many-place copulas connect plural logical subjects to logically non-distributive predicates that denote universals that have scattered exemplification.*

2 Weakly Internal Relations and Scattered Exemplification

I want to discuss relations such as *taller than*, *larger than*, *heavier than*, *warmer than*, and *brighter than*. All of them conform to the ordinary Armstrongian definition of internal relations, which says: the relation *R* in *aRb* is internal if and only if, necessarily, *if* both *a* and *b* exist, then *R* exists (Armstrong 1997, p. 87). According to this definition, however, even the exemplification relation is internal. But, at least for the purposes of this paper, they should be kept distinct. Therefore, I will introduce a distinction between two kinds of internal relations: strongly internal and weakly internal, respectively. The definitions look as follows (Johansson 2011a):

- There is between *a* and *b* a strongly internal relation iff, *a* cannot exist if *b* does not exist, *and/or* vice versa.
- There is between *a* and *b* a weakly internal relation *R* iff, *a* and *b* can exist independently of each other, but *if* both exist then, necessarily, *aRb*.

The exemplification relation is strongly internal (the exemplification cannot exist without the universal), but the others mentioned are weakly internal. This dot ● contains an exemplification of blackness (*b*), but if there were no universal blackness (*B*), the exemplification *b* (and the state of affairs *BEb*) would not exist. A relation such as *larger than*, on the other hand, has relata that can exist independently of each other. If the dots within the brackets [● •] are called *a* and *b*, respectively, then it is true that *a* and *b* can exist independently of each other, but it is also true that, given both of them, the relation *larger than* must be there.

The strongest argument I know against the existence of mind-independent weakly internal relations consists (applied to *larger than*) in the question below and a rejection of all the four at first seemingly possible answers (Heil 2005, 2009). The question is:

Where is the *larger than* relation of the state of affairs within the brackets [● •]?

And the answers to consider are: (i) in *a*, (ii) in *b*, (iii) in both *a* and *b*, and (iv) somewhere else. However, one might well argue:

(i´) the relation cannot possibly be in *a* alone, since it is a relation, i.e., not possibly: ^{is}Ra;

(ii´) for the same reason, it cannot be in *b* alone, i.e., not possibly: ^{is}Rb;

(iii´) nor can it be in both *a* and *b*, since it is only one single entity;

(iv´) it cannot be exemplified anywhere else, since *a* and *b* are the only possible relation bearers around.

I regard the statements (i´), (ii´) and (iv´) as indisputable. If there is a mind-independent weakly internal relation *larger than* that is exemplified within the square brackets above, then it is statement (iii´) that has to be questioned. If *larger than* is represented by *R*, and the symbolism I introduced at the end of the former section is allowed, then statement (iii´) can be given three different representations:

(iii´ a) not possibly: ^{is}Ra and ^{is}Rb

(iii´ b) not possibly: $^{is}R(a$ and $b)$

(iii´ c) not possibly: $a\ ^{is}R^{is}\ b$.

The statements (iii´ a) and (iii´ b) contain only the ordinary one-place copula, whereas statement (iii´ c) contains the two-place copula. Let us now look at the three alternatives one by one.

Since I have already said that both the answers (i´) 'not possibly: ^{is}Ra' and (ii´) 'not possibly: ^{is}Rb' are indisputable, I do of course regard the statement (iii' a), 'not possibly: ^{is}Ra and ^{is}Rb', as indisputable, too. The word 'and' is here identical with the ordinary logical conjunction sign, and it cannot represent anything that would turn the impossibilities stated in the conjuncts into a possibility for the conjunction as a whole.

In (iii´ b), as in (iii´ a), the copula represents the ordinary exemplification relation, but 'and' cannot here be the ordinary logical conjunction sign; the expression '$(a$ and $b)$' represents the collection (or aggregate or mereological sum) of *a* and *b*. In the statement '$^{is}R(a$ and $b)$' it is claimed that the relational universal *larger than* is exemplified in the ordinary way in the collection of the particulars *a* and *b*; moreover, in the collection *as one*. Otherwise we are back in alternative (iii´ a). But if *larger than* is exemplified in the collection as one, it is a monadic property of the collection, not a relation between the members of the collection, i.e., not a relation in the collection *as many*. Therefore, if *R* is a weakly internal relation, then statement (iii´ b) holds true: 'not possibly: $^{is}R(a$ and $b)$'.

(As can be seen, the distinction between a collection as one and as many is important, and I will use it a couple of times in the rest of the paper, too. I take it to be more or less the same distinction as that between classes as one and classes as many that Russell once made (2006, §70). About this distinction and its history, see (Simons 1982, §1).)

What then are we to say about the remaining alternative, (iii´ c), which employs a two-place copula? If a two-place copula should be of ontological interest in the cases under discussion, it must be able to represent a special kind of exemplification: *scattered exemplification*. In the case at hand there is 2-scattered exemplification; a three-place copula will represent a 3-scattered exemplification relation, and so on. This relation of scattered exemplification must not be conflated with the *scattered localizations* of the non-scattered exemplifications of monadic property universals. Scattered exemplification of a relation, REp_n, means that one single exemplification relation (E) has a spatiotemporal scatter (p_n) as one of its relata, and a relational universal (R) as the other. In the state of affairs [● •], there is in my view a mind-independent relational universal *larger than* that is directly exemplified in the plurality of the collection of the two dots.

I admit that what I have said at first sounds odd (at least it did so to me when I started to think about it), but my problem is that I find it equally odd, or even more odd, to deny mind-independent reality to all these weakly internal relations that I have mentioned. They play an essential role in the natural sciences. To argue in favor of scattered exemplifications and concomitant many-place copulas, must be, I think, like it was – once upon a time – to argue for the existence of at first seemingly absurd entities such as irrational numbers (think of the Pythagoreans) and non-Euclidean geometrical structures (think of geometricians before the 19th century). For a while, one has simply to assume the existence of the entities, start to talk about them, think of all kinds of possible consequences, and check whether any contradictions appear. If not, the entities in question can (and probably will) be accepted. So far, I haven't found any contradictions when thinking about scattered exemplification. Here are two brief lines of thought in favor of the possibility of exemplification relations with a spatial scatter as its relata on the spatiotemporal side.

First, if 2-scattered exemplification really is a matter of fact, then the critics' question where the relation *larger than* is to be found rests on a category mistake. Their question 'where *is* the relation?' should be exchanged for 'where *is—is* the relation?' And then it does not seem odd to give the answer 'in the plurality of the collection of *a* and *b*'. In other words, if the question where to find the ^{is}*larger-than* relation is exchanged for the question where to find the $^{is\text{-}is}$*larger-than* relation, then we can answer: in the collection of *a* and *b* as many.

Second, if 2-scattered exemplification exists, then it is no mystery, but a natural outcome of our spatiotemporal life situation, that everyday language is replete with talk about states of affairs that are disconnected in space: 'the Sun is larger than the Earth' ('the Sun ^{is}*larger-than*^{is} the Earth'), 'Simmias is taller than Socrates' ('Simmias ^{is}*taller-than*^{is} Socrates'), etc. We talk this way because, thereby, we can talk about and discuss mind-independently existing exemplifications of weakly internal relations.

3 Language, Logic, Plural Predication, and Many-Place Copulas

When I wrote "Hypo-Realism with Respect to Relations" I was not aware of books such as H. Ben-Yami, *Logic & Natural Language: On Plural Reference and Its Semantic and Logical Significance* (2004) and T. J. McKay, *Plural Predication* (2006), and the corresponding journal discussions about plural reference and non-distributive predication in natural languages and logic; for an early presentation of some of the problems see (Simons 1982, §2). Let me briefly introduce the most essential notions.

Ordinary English contains many *plural subject terms*: 'we', 'you', 'they', 'these Xs', 'those Xs', 'the Xs', '*a*, *b*, and *c*', etc. When used, each such term has at least an attempted *plural reference*. In a predication with a plural subject term, normally, the auxiliary verb used is 'are'. However, there are two different kinds of such *plural predications*: distributive and non-distributive. In a *distributive* plural predication the predicate can be applied to each of the referents separately. Without loss of content, the sentences to the left below can be substituted by those to the right:

'They (2 persons) are tall' = '*a* is tall and *b* is tall'
'They (5 persons) are wearing hats' = '*a* is wearing a hat, *b* is wearing a hat, *c* is wearing a hat, *d* is wearing a hat, and *e* is wearing a hat'
'They (100 persons) are in the house' = '*a* is in the house, *b* is in the house, etc.'

In a *non-distributive* plural predication, on the other hand, the predicate cannot be applied to each of the referents separately. If one wants a substitution of 'they', it must be by a new plural subject term such as '*a*, *b*, and *c*':

'They (2 persons) are brothers' = '*a* and *b* are brothers'

‘They (5 persons) are lifting the piano’ = ‘*a*, *b*, *c*, *d*, and *e* are lifting the piano’

‘They (100 persons) are surrounding the house’ = ‘*a*, *b*, *c*, etc. are surrounding the house’

On the left hand side in both the distributive and the non-distributive examples, the same plural subject term (‘they’) and the same auxiliary verb term (‘are’) is used. Therefore, the difference must be somewhere in the predicates. And it is easy to find: ‘brother’ can by definition not be a monadic predicate; persons with normal capabilities cannot lift a piano alone; and one human being cannot possibly surround a house. Therefore, in these cases, the plural predication can only be non-distributive, and the term ‘are’ cannot possibly be substituted by an iteration of the term ‘is’.[3]

My proposal is that the ‘are’ of the plural distributive predications should from a logical point of view be seen as representing a one-place copula, whereas the ‘are’ of the plural non-distributive predications should be seen as representing a many-place copula. Non-distributive predication is, as McKay says, “a routine part of ordinary language use, yet standard systems of first-order logic provide no place for such non-distributive predication (ibid., p. 2).” He tries to remedy this fact by extending first-order logic, but when doing this he still clings to the rule that predicate logic should contain no copula symbols. Therefore, his book does not lead to any discussions about the possibility of many-place copulas and scattered exemplifications. And the same, even though for other reasons, goes for Ben-Yami. He retains a copula, but seems to miss the possibility of many-place copulas because he does not pay any special attention to non-distributive predications; he focuses almost exclusively on plural reference.

In my symbolism, the six predication examples above can (with T for ‘tall’, B for ‘brother’, H for ‘wearing hats’, L for ‘lifting a piano’, I for ‘in a house’, and S for ‘surrounding a house’) be written as below. If the predicates are interpreted as referring to universals, then the predicates used in distributive predication refer to universals that have the traditional exemplification relation, whereas the predicates used in non-distributive predication refer to universals with scattered exemplification.

$^{is}Ta \wedge {}^{is}Tb$	2-term distributive predication
$^{is\text{-}is}Bab$ (or: $a\ {}^{is}B^{is}\ b$)	2-term non-distributive predication
$^{is}Ha \wedge {}^{is}Hb \wedge {}^{is}Hc \wedge {}^{is}Hd \wedge {}^{is}He$	5-term distributive predication
$^{is\text{–}is\text{–}is\text{–}is\text{–}is}Labcde$	5-term non-distributive predication

$^{is}Ia \wedge {}^{is}Ib \wedge {}^{is}Ic \wedge \ldots$	100-term distributive predication
$^{is–is–is–\cdots}Sabc\ldots$	100-term non-distributive predication

The wearing-a-hat case is above classified as one of distributive plural predication. However, the five persons in question could for a joke's sake be wearing a special hat that takes in all five heads. If so, the predication would be non-distributive. This shows that some predicates can be used in both a distributive and a non-distributive sense. If such distribution-ambiguous predicates are left aside, then the notion of 'distributive predicate' can be defined as follows:

> *F* is *distributive* if it is analytic that *F* is true of some things iff it is true of each of them separately (Oliver & Smiley 2008, p.23)

Of course, a predicate *F* is non-distributive if it is not distributive. The philosophers quoted, however, do not call the negation 'non-distributive' but 'collective'. And this is a mistake. There are different kinds of non-distributive predicates, and all of them do not deserve the label 'collective'.

The predicates earlier mentioned in this section are collective predicates: *together, a* and *b* are brothers; together, *a*, *b*, *c*, *d*, and *e* are lifting the piano; and together, *a*, *b*, *c*, etc. are surrounding the house. It sounds somewhat odd, but one may perhaps also say that 'together, *a* and *b* are equally tall', or 'together, *a* and *b* are unequally tall'. The same thing, however, cannot possibly be said, about the *asymmetric* internal relations that I mentioned in the first two sections; a fact that is very much also the reason why Russell started to regard relations as irreducible entities, see (Russell 2006, §§ 208–216). Two particulars *a* and *b* cannot together be *taller than*, *larger than*, *heavier than*, *warmer than*, and *brighter than*. It is *one* of the particulars that has an asymmetric relation to *the other*. Nonetheless, sentences such as '*a* is taller than *b*' contain, when '*a* and *b*' is regarded as a single plural subject term, a non-distributive plural predication. Therefore, non-distributive predicates should be divided into at least two kinds: collective predicates and asymmetric relation predicates, respectively. Both kinds of predicates, however, do via the exemplifications of the universals they refer to denote a collection as many.

As far as I can see, my idea of invoking many-place copulas in cases of non-distributive predication is no more curious than McKay's idea of plural non-distributive predications

without a copula. And if such many-place copulas are allowed, then we can construe sentences whose default ontologization posits scattered exemplification.

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[1] It should perhaps be noted that Russell nonetheless in his popular *The Problems of Philosophy* says that universals do not exist in the same sense as things do: "we shall say that they *subsist* or *have being*, where 'being' is opposed to 'existence' as being timeless (Russell 2001, p. 57)." In his *The Philosophy of Logical Atomism*, however, he rejects such a way of talking see 1989, pp. 255-256). Since many philosophers think that universals and particulars must have different modes of being, it is worth noting that Russell ended up by believing that particulars are only complexes of universals (Russell 1962). Once he says that the existential quantifier is to the existence of things, properties, and relations as the genus fish is to the different species of fish (Russell 1959, pp. 231–238). To the view that the modalities possibility, actuality, and necessity must be different modes of being, he retorts that those who think in this way have not understood his distinction between propositions and propositional functions; primarily, these modalities apply only to propositional functions, and such functions are neither true nor false (Russell 1989, chapter V).

[2] Note that a *relational property predicate* such as 'taller than Socrates' needs only the traditional one-place copula, 'Simmias *is* (taller than Socrates)', but a two-place *relation predicate* such as 'taller than' needs a two-place copula.

[3] The expressions 'are brothers', 'are lifting the piano', and 'are surrounding the house' can of course be used in relation to a varying number of persons. For a defense of so-called 'multigrade' or 'variably polyadic' predicates, see (Oliver & Smiley 2004).

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