Metrological Thinking Needs the Notions of *Parametric* Quantities, Units, and Dimensions

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Abstract

The paper introduces a distinction between true metric and parametric quantities, units, and dimensions. It claims in its first part that these distinctions shed important light both on the base quantity amount of substance and VIM's definitions of kind of quantity and quantity dimension. The second part is devoted to the unit one, and it claims that this unit must be regarded as a parametric unit, but also that it is not as needed as has been thought. The third part takes for granted that both the mole and the unit one are parametric units, and it argues that, for pedagogical reasons, the mole should be exchanged for the unit one, and the parametric quantity amount of substance be renamed to 'elementary entities'.

The metrologically new and central term of this paper is 'parametric'. What the term is intended to mean should be clear from the way it is used in the paper, but some introductory words might make the reading easier. The prefix 'para-' has here the meaning of *beyond*. Parametric quantities and units are quantities and units that in a certain sense go beyond the ordinary, so to speak true, metric quantities and units used in measurements. Although the parameters spoken of are not parameters for numbers or quantity values, there is a kinship with the mathematical notion of parameter that is used when in the function '$y = kx$' k is called a parameter. Before k is given a specific value, the function does not represent any specific straight line in the x-y coordinate system; and before parametric quantities and units are specified, they cannot give rise to any meaningful physical-chemical comparisons. The concept of comparison now introduced is, just like in the third edition of VIM (henceforth VIM3) [1], treated as a non-defined primitive concept.

The claim of this paper is that metrology would profit from distinguishing between true and parametric quantities, units, and dimensions. In particular, these distinctions have repercussions on how to look at the unit one, the mole, and the corresponding quantities and dimensions.

PART I: Parametric Quantities, Units, and Dimensions

1. The Peculiarity of the Unit Mole and the Quantity Amount of Substance

The introduction of the base unit mole in the SI brochure differs in structure from the introductions of all the other six base units of the SI system; it contains two paragraphs, the others only one (corresponding to the first paragraph below). The brochure says:

> 1. The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is “mol”.
> 2. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles. [2, p. 115]

This means that, strictly speaking, the base quantity at hand is not just amount of substance, but *amount (of substance) of elementary entities of a certain kind*; briefly, amount(-of-substance)-of-E_p, where the subscript p functions as a parameter whose “values” are atoms, molecules, ions, electrons, etc. Therefore, the quantity amount of substance had better be called a *parametric* quantity. The main reason behind the requirement of paragraph 2 is, I take it, that it makes no physical-chemical sense to compare amounts of different kinds of elementary entities; and comparisons of quantity values are what make a quantity a true metric quantity, i.e., a practically useful and interesting quantity. But to compare an amount of atoms with an amount of molecules would be like comparing a number of houses with a number of blocks.

When the elementary entities spoken of in the definition become specified, then even the mole unit becomes specified. That is, the term ‘mole’ has in practice, ever since its introduction in the SI (1971), been used as if it means not just mole but mole-of-E_p. The mole is not a base unit on a par with the six property base units; it cannot be used in significant physical-chemical comparisons until the subscript-parameter p in mole-of-E_p has been given a certain “value.” In analogy with the notion of ‘parametric quantity’, I will call the *unspecified* mole a ‘*parametric* unit’. Without a specification of what kinds of elementary entities expressions such as ‘2 mole’ and ‘5 mole’ refer to, these expressions cannot in relation to each other symbolize any meaningful physical-chemical comparison.

If only for the reason now given, I think the SI system would become clearer if the difference between the mole and the other base units was stressed by calling the mole a ‘parametric unit’ (the true units may well be called only ‘units’); and that the difference between amount of substance and the six property base quantities was stressed by calling amount of substance a ‘parametric base quantity’. But there is more to be said in favor of an introduction in metrology of these notions.

2. VIM3:s Definitions of Kind-of-Quantity and Quantity Dimension

My critical remarks on VIM3 in this section has affinities with some already put forward in *Metrologia* by L Mari [3], but my proposals for terminological improvements are not the same as his. Before bringing in the opposition ‘parametric x’ — ‘(true metric) x’, some other distinctions have to be made perfectly clear. VIM3 starts with a definition of *quantity* (def. 1.1) followed (1.2) by one for *kind-of-quantity* (the hyphenation is added by me, but aligns with R Dybkaer [4]). Two other VIM3 definitions relevant for my remarks are those of *quantity dimension* (1.7) and *quantity value* (1.19). Let me start with quantity:

> **1.1 quantity** = property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference [1, p. 2].

Unfortunately for the terminological simplicity of this paper, this definition does not supply us with the term ‘quantity’ that appears in the eighth edition of the SI brochure [2] (henceforth SI8), which I have used in Section 1 and I will use again in Sections 3-6. Since phenomena and bodies are spatiotemporally localized individual entities, so are their property instances. That is, VIM3 supplies us, more precisely, with a definition of *individual quantity* (examples: specific length instances, specific mass instances, etc.). In the SI8, on the other hand, the natural interpretation of expressions such as ‘base quantity’ and ‘derived quantity’ is that ‘quantity’ here means quantity in a general sense; rather, it corresponds to what VIM3 calls ‘kind-of-quantity’ (see below). Therefore, I will for clarity’s sake add a parenthesis and in what follows call VIM3’s quantity ‘(individual) quantity’.

VIM3 then defines *quantity value* as an expression that is related to a spatiotemporally localized (individual) quantity, i.e., a quantity value is a *representation* of a (individual) quantity. The definition says:

> **1.19 quantity value** = number and reference together expressing magnitude of a (individual) quantity [1, p. 12, the parenthesis added].

It is explicitly said that the quantity value 5.34 m is the length of *a given rod*, and that the quantity value 0.152 kg is the mass of *a given body*. Therefore, for clarity’s sake, I will in what follows call VIM3’s quantity value ‘quantity value (of individual)’.

The SI8 brochure does not use the very term ‘quantity value’, but since 1 m is defined as something general (the length of the paths travelled by any light beam in vacuum during a certain specified time interval), the length 5.34 m becomes implicitly defined as being a *general* quantity value. Of course, in contradistinction to 1 m, 1 kg is not yet given a theoretical and general definition; it is still defined by means of the mass instance of the prototype at BIPM in Paris. Nonetheless the term ‘1 kg’ (and by implication ‘0.152 kg’) has a meaning that makes it refer to a *general quantity value*. Why? Answer: because it is tacitly taken for granted that all the mass instances that are exactly similar to that of the prototype have the mass 1 kg, too. If this was not the case, it would be impossible ever to substitute the prototype by a general theoretical definition.

When we speak of the quantity values of *the* meter scale and *the* kilogram scale, we are not talking about the quantity values (of individuals) of VIM3, but of general quantity values. If, to take an example, each of five different rods is 5.34 m long, then each can be ascribed the quantity value (of individual) 5.34 m, but the abstract meter scale must be ascribed the general quantity value 5.34 m. When SI8 says that one of the tasks of the BIPM is to “establish fundamental standards and scales for the measurement of the principal physical quantities [2, p. 95],” it talks about scales in the abstract sense; scales whose values are general quantity values.

Whereas SI8 in effect treats quantities and quantity values as being general, VIM3 defines quantities and quantity values as being or expressing, respectively, individual spatiotemporally localizable entities. In my opinion, both the SI brochure and the VIM document ought more explicitly than is done in SI8 and VIM3 spell out all the distinctions of the fourfold matrix below (some terms explained afterwards):

	Quantity	**Quantity value**
Individual	**(individual) quantity** (quantity instance)	**quantity value (of individual)** (representation of a (individual) quantity)
General	**general quantity** (a class of quantity instances *or* a universal)	**general quantity value** (representation of a general quantity)

Matrix 1.

In passing, I would like to make a brief remark on the question what terms for general quantities might refer to. In philosophy there are two main answers. So-called 'realists', Platonists in their way and Aristotelians in theirs, claim that each of the terms refer to a *universal*, whereas so-called 'nominalists', both class nominalists and resemblance nominalists, claim that each refers to a *class* of instances; more precisely, an equivalence class. (For this philosophical terminology, see D Armstrong [5].) Mari, for example, brings talk about classes into metrology [3, sec. 2]; and so do today many people concerned with metrological issues in informatics and computer science.

Let us next look at VIM3's definition of kind-of-quantity. According to Mari, and to some extent I agree, it creates a conceptual inconsistency in VIM3; length for example appears both as a (individual) quantity and as a kind-of-quantity [3, p. L11]. I will, however, in my own way make the terminological improvements that I think are needed in order to set everything right. Here is the definition:

> **1.2 kind-of-quantity** = aspect common to mutually comparable quantities [1, p. 3].

The definition may at first sight be regarded as being ambiguous between being a definition of something individual and of something general. If 'quantity' in kind-of-quantity is understood according to VIM3's definition of quantity, then it should be a definition of *individual* kind-of-quantity, but since the defining expression refers to something (an aspect) that different individual quantities have *in common*, it should be a definition of something general. However, since normally the defining expression takes precedence over the defined expression, I will regard a kind-of-quantity as being something general. This interpretation is also supported by the following facts: (a) in Note 1 to the definition, length and energy in the general sense figure as examples of kinds-of-quantities; (b) in definitions such as those of measurement unit (1.9), ordinal quantity (1.26), and quantity-value scale (1.27) the term 'kind-of-quantity' is definitely used as if it means *general* kind-of-quantity. Therefore, in what follows, I will call VIM3's kind-of-quantity '(general) kind-of-quantity'.

The aspect talked about is then what is general (common) both in two different (individual) quantities that have the same general quantity value, and in different (individual) quantities that have different general quantity values such as 1 m, 5.34 m, and 13 km. This implies that even the different corresponding *general* quantities have the same (general) kind-of-quantity. I take it for granted that general quantities inherit comparability from the corresponding (individual) quantities; direct physical-chemical comparability can only exist between spatiotemporal individuals.

Now, even though VIM3's notion of 'kind-of-quantity' must mean *general* kind-of-quantity, there is no problem in introducing a notion of '*individual* kind-of-quantity', too. Where there is talk of a (individual) quantity, there can be talk of an *individual* kind-of-quantity, too; where there is said to be a rod of 5.34 m, there can be said to be an instance of the (general) kind-of-quantity length, too. Matrix 1 above can be expanded into Matrix 2 below (some terms explained afterwards):

	Kind-of-quantity	**Quantity**	**Quantity value**
Individual	**individual kind-of-quantity**	**(individual) quantity**	**quantity value (of individual)**
General	**(general) kind-of-quantity** (a class of general quantities *or* a determinable universal)	**general quantity** (a class of quantity instances *or* a determinate universal)	**general quantity value**

Matrix 2.

Since VIM3 does not use the notions of 'individual kind-of-quantity' and 'general quantity', one might say that it hides the distinction between the general and the spatiotemporally individual in its distinction between quantity and kind-of-quantity; the added parentheses uncover this fact.

Note that the possible classes referred to by ‘general quantity’ have as their members or objects instances, whereas the classes referred to by ‘(general) kind-of-quantity’ have as members or objects not instances but the aforementioned *classes*. A requirement, not mentioned in the matrix, is that a class of classes that constitutes a (general) kind-of-quantity must contain an ordering relation. Furthermore, classes of classes must not be conflated with so-called ‘superclasses’; Mari [3, sec. 2]. If what is general is taken to be universals not classes, then a distinction between determinable and determinate universals (see [6]) substitutes the distinction between classes of classes and classes.

Matrix 2 contains an obvious asymmetry. It has one column for Quantity and one for representations of quantities, the column Quantity value, but there is no column for representations of what can be found in Kind-of-quantity. As Quantity values express and represent Quantities, there should be something that express and represent Kinds-of-quantity, too; as a Quantity (individual or general) *has* a certain Quantity value, there should be something that (in the same sense of ‘has’) a Kind-of-quantity *has*. The closest one comes to such a representational entity in VIM3 is what it calls ‘quantity dimension’. Kinds-of-quantities are said to *have* a quantity dimension, even though this verb does not appear in the very definition:

> **1.7 quantity dimension** = expression of the dependence of a quantity on the base quantities of a system of quantities [1, p. 4; the def. is shortened].

In its first example of a quantity dimension, VIM3 says that “the quantity dimension of force is denoted by dim F = $\mathsf{L\ M\ T}^{-2}$ [2, p. 4].” As far as I can see, this seemingly simple sentence must mean something quite complicated; namely that the quantity dimension of force is *referred to* (denoted) by the expression ‘dim F’, and *is* the symbol list $\mathsf{L\ M\ T}^{-2}$, which, in turn, *expresses* how force is dependent on the SI base quantities. Since force is a (general) kind-of-quantity, the term ‘quantity’ in the defining expression of def. 1.7 does on this interpretation cover both VIM3’s term ‘(individual) quantity’ and its term ‘(general) kind-of-quantity’. This interpretation of mine is clearly supported in a second example, where it is explicitly said that $\mathsf{M\ L}^{-3}$ *is* the dimension of mass density.

Thus the quantity dimension of length *is* the symbol L (referred to by ‘dim L’), and that of mass *is* the symbol M (referred to by ‘dim M’). Being base (general) kinds-of-quantity, length and mass are dependent only on themselves. Therefore one can say that the (general) kind-of-quantity length *has* the quantity dimension L, and that that of mass *has* the quantity dimension M, just as that of force *has* the quantity dimension L M T^{-2}. Consequently, Matrix 2 can be expanded into Matrix 3 below; the columns of Kind-of-quantity and Quantity contain physical-chemical entities, whereas the columns of Quantity dimension and Quantity value contain representational entities; this is meant to be indicated by the arrows in these columns. (Later, Section 3, I will discuss a possible notion of 'dimension' that does not, like VIM3's 'quantity dimension', identify a dimension with a mere list of symbols.)

	Kind-of-quantity	**← Quantity dimension**	**Quantity**	**← Quantity value**
Individual				
General				

Matrix 3.

Now, at last, it is time to present the observation of VIM3 that for me actualizes the distinction between *parametric x* and *true metric x*, and which I think implies that the term 'Quantity dimension' in Matrix 3 should be read as meaning *true* quantity dimension. Sometimes there is a one-to-one relationship between a specific quantity dimension and a specific kind-of-quantity, but sometimes there is a one-to-many relationship. VIM3 observes:

> NOTE 2 Quantities of the same kind within a given **system of quantities** have the same **quantity dimension.** However, quantities of the same dimension are not necessarily of the same kind.
> EXAMPLE The quantities moment of force and energy are, by convention, not regarded as being of the same kind, although they have the same dimension. Similarly for heat capacity and entropy, as well as for number of entities, relative permeability, and mass fraction [1, p. 3].

Of the examples here mentioned, I will in this section discuss only the first two, moment of force in relation to energy and heat capacity in relation to entropy. The other examples are ascribed the dimension one, and will be discussed later; relative permeability and mass fraction in Section 3, and number of entities in Sections 4-5.

In a note preceding the last quotation, VIM3 says that the terms 'heat', 'kinetic energy', and 'potential energy' refer to the same (general) kind-of-quantity, energy. This view is in conformance with SI8's statement (italics added): "In order to establish a system of units, such as the International System of Units, the SI, it is necessary first to establish a system of quantities, including a set of *equations defining the relations between those quantities* [2, p. 3]." Since, in classical physics, heat, kinetic energy, and potential energy (all measured in joule, J), can be transformed into each other and into work (= force times distance; N m = J), they can in a significant physical-chemical way be *compared*, and are so (according to VIM3's definition 1.2) of the same (general) kind-of-quantity. By implication, they also have the same quantity dimension, that of energy: $\mathsf{L}^2\ \mathsf{M}\ \mathsf{T}^{-2}$.

On the other hand, the (general) kind-of-quantity moment of force (for short: torque, i.e., force that causes rotation times perpendicular distance to the rotation center; N m = J [2, p. 119]) has no such comparability relationship to heat, kinetic energy, potential energy, or any other energy form; even though it has the quantity dimension $\mathsf{L}^2\ \mathsf{M}\ \mathsf{T}^{-2}$ (force times distance), too. Despite being measured by the same unit, J, there is no significant physical-chemical way in which energy and torque can be compared with respect to magnitude. And the same goes for heat capacity and entropy, whose common quantity dimension is $\mathsf{L}^2\ \mathsf{M}\ \mathsf{T}^{-2}\ \Theta^{-1}$, and common measurement unit is joule/Kelvin; Θ is the quantity dimension of thermodynamic temperature. (Similar remarks can be found in W H Emerson [7].)

Some quantity dimensions, e.g., L and M, have a one-to-one relationship to a (general) kind-of-quantity, whereas others such as $\mathsf{L}^2\ \mathsf{M}\ \mathsf{T}^{-2}$ and $\mathsf{L}^2\ \mathsf{M}\ \mathsf{T}^{-2}\ \Theta^{-1}$ have a one-to-many such relationship. In my opinion, this difference should be stressed even more than is done in VIM3. The former (one-to-one) kind of quantity dimensions should be regarded as *true* quantity dimensions, and the latter (one-to-many) kind of dimensions should be called '*parametric* quantity dimensions'. The metrologically important difference is this: all the possible quantity values (of individuals) that can be connected to a true quantity dimension are physical-chemically comparable, but all the possible quantity values (of individuals) that can be connected to a parametric quantity dimension are not, since, by definition, they can bring in more than one (general) kind-of-quantity.

What now about the quantity dimension (in VIM3's sense) of amount of substance, i.e., N. From what has already been said in Section 1, it follows that N must be a parametric quantity dimension. Everything that can be measured in terms of the mole is not physical-chemically comparable. On the other hand, everything that can be measured in terms of a certain mole-of-E_p is physical-chemically comparable. This has an interesting consequence, not noted in VIM3. According to VIM3's definition of (general) kind-of-quantity (def. 1.2), *amount of* *substance* is not a (general) kind-of-quantity, but *amount (of substance) of elementary entities of kind* E_p is.

The last observation gives rise to a new question: what should amount of substance be called in VIM3 terminology? Since it is not an individual quantity (nor, by the way, a general quantity), not a general kind-of-quantity (nor, by the way, an individual kind-of-quantity), and not a quantity dimension (either general or individual) – what is it? As far as I can see, the best solution is (a) to bring in again the opposition between being a parametric x and a true x, (b) let VIM3's term '(general) kind-of-quantity' be short for '*true* (general) kind-of-quantity' and (c) call amount of substance a '*parametric* (general) kind-of-quantity'. Put bluntly, this is in essence the same distinction as that made in Section 1 in terms of 'true quantity' and 'parametric quantity'.

Amount of substance ought in the SI8 brochure be called a 'parametric quantity' and in the VIM3 document a 'parametric (general) kind-of-quantity'; its quantity dimension N ought in the VIM3 be called a 'parametric quantity dimension', and its unit, the mole, ought in both documents be called a 'parametric unit'.

Let me now move back again to the (general) kinds-of-quantity moment of force, energy, heat capacity and entropy. A parametric unit is a unit that needs a specification before it can give rise to physical-chemically meaningful comparisons. According to this characterization, the units joule and joule/kelvin must outside of specific contexts be deemed to be parametric units. Energy-joules cannot in a significant way be compared with torque-joules, and heat-capacity-joule/kelvin not with entropy-joule/kelvin. Put more generally, if two different true (general) kinds-of-quantities (K_1 and K_2) are ascribed the same measurement unit, then this unit is a parametric unit. What the mole is to mole-of-E_p, the joule is to joule-of-K_p, and the joule/kelvin is to joule/kelvin-of-K_p; in the joule case, p can take at least the "values" energy-joule and torque-joule, and in the joule/kelvin case at least the "values" heat-capacity-joule/kelvin and entropy-joule/kelvin.

This being noted, it is time to present the fourth and last matrix:

	Parametric kind-of-quantity	← Parametric quantity dimension	(True) kind-of-quantity	← (True) quantity dimension	Quantity	← Quantity value
Individual						
General						

Matrix 4.

That 'True' is put within parenthesis means, as the parentheses in matrixes 1 and 2 do, that if you delete the whole parenthesis you will get the corresponding VIM3 term. This last matrix makes use of all the four abstract distinctions listed below. As far as I can see, all four are needed if one wants to make everything in basic metrology explicitly clear; the first three are not enough.

1. general—individual (alternatively: universal—particular and class—member-of-class); the two rows in the matrix.
2. representation—what-is-represented (alternatively: expression—what-is-expressed and symbol—what-is-symbolized); indicated by the arrows in the matrix.
3. class—class-of-classes (alternatively: determinate—determinable); the column Quantity in relation to Kind-of-quantity.
4. parametric—true-metric; column one in relation to column three, and two in relation to four.

VIM3 takes it for granted, quite rightly in my opinion, that there are in the language- and symbol-independent part of the world property instances, some with and some without a magnitude, and that Quantity values only belong to a representational medium. According to my experience, if the distinction representation—what-is-represented is not only present in a context, but is itself in this context explicitly represented (as it is and should be in VIM), then often many conceptual and terminological complications follow in the wake. Therefore, I am not especially astonished over the amount of complexity that is unfolded in Matrix 4.

3. Unit One, Dimension One, and Dimensionless Quantities

In this section, after some important preliminaries, I will discuss cases where derived quantities in the SI8 sense are allotted the measurement unit one. VIM3's examples (see quotation in Section 2) are relative permeability and mass fraction, but for the sake of simple examples I have substituted relative permeability by relative length. (The SI8 notion of 'quantity' becomes when translated into Matrix 4 terminology the notion: 'general, true or parametric, kind-of-quantity'.)

In SI8, the terms 'dimension' and 'quantity' tend to become synonymous. Each base quantity "is regarded as having its own dimension," and the dimension of a derived quantity is said to contain the same information about its relation to the base quantities as that "provided by the SI unit of the derived quantity as a product of powers of the SI base units" [2, p. 105]. Probably, this identification tendency is due to the fact that only quantities are meant to be discussed. But I think it is important to keep the terms distinct; one reason being that the concept of 'dimension' (not to be conflated with VIM3's 'quantity dimension') is in a specific sense logically prior to that of 'quantity'. Every quantity has a dimension, but there may be physical-chemical dimensions that do not lend themselves to quantification; there is no metaphysical truth to the effect that all physical-chemical dimensions can be quantified. Shape, for instance, which plays quite a prominent role in science and technology, has not yet been quantified despite the fact that all shapes are comparable. The comparability is shown by the fact that two arbitrary determinate shapes can always, by a more or less continuous chain of intermediate shapes, be connected; such a connection is not possible to create between a shape and, on the other hand, a color, a mass, an electric charge, etc [6, sec. 3]. Therefore, a quantity is not simply a dimension; it is a *quantified dimension*. Length and weight were assumed to be dimensions before they became quantified dimensions.

(In the terminology of VIM3, spatiotemporally localized determinate shapes should, since they lack a magnitude, be called '(individual) nominal properties'. Words such as 'circular' and 'star-shaped' could then be called (individual or general) 'nominal property values for shape'. Shape as a counterpart to 'kind-of-quantity' could be called a 'nominal kind-of-property', and the symbol S could be regarded as being the nominal property dimension of shape.)

Shape is because of the comparability of all the determinate shapes a true dimension, but since it has not been quantified it lacks a unit. Necessarily, an un-quantified dimension is also a *unit-less dimension*. Soon, I will argue that even some derived quantities should be regarded as having a unit-less dimension. But, first, what more exactly is a dimension in the sense now at issue?

At least three different answers that catch something like a common core can be given. First, if the notion '(general) kind-of-quantity' is taken as unproblematic, a dimension can be said to be what is obtained when from a certain (general) kind-of-quantity *all* ordering relations and measurement units are abstracted away. Second, if the notion of 'property in the general sense' is taken as unproblematic, a dimension is what all physical-chemically comparable general properties have in common; be these properties in principle quantifiable or not. Third, if only the notion of 'property instance' is taken as unproblematic, as a first step, a property in the general sense can be defined as (in the terminology of logic) an *equivalence class* of real and possible property instances; and then a dimension can be defined as a *class of classes* of all the equivalence classes that constitute comparable general properties. According to this third view, the four general quantity terms '1.7 kg', '13.82 kg', '99 kg', and '1000.001 kg' refer to four different equivalence classes of real and possible mass instances (each class being based on a relation of exact similarity), and the term 'dim *M*' refers to the class of all possible such equivalence classes.

If dimensions are understood in this sense, the distinction between parametric and true quantities made in Section 1 is automatically mirrored by a distinction between parametric and true dimensions. Amount of substance, for instance, is not only a parametric quantity, it is a parametric dimension, too; and so is, I will next argue, the dimension one.

About derived quantities and dimensions, the SI8 brochure makes the following general statement (the italics for whole words are added):

> The dimensions of the derived quantities are written as products of powers of the dimensions of the base quantities using the equations that relate the derived quantities to the base quantities. In general the dimension of any quantity *Q* is written in the form of a dimensional product,
>
> $$\dim Q = \mathsf{L}^{\alpha}\,\mathsf{M}^{\beta}\,\mathsf{T}^{\gamma}\,\mathsf{I}^{\delta}\,\Theta^{\varepsilon}\,\mathsf{N}^{\zeta}\,\mathsf{J}^{\eta}$$
>
> where the exponents α, β, γ, δ, ε, ζ, and η, which are generally small integers which can be positive, negative or zero, are called the dimensional exponents. The

dimension of a derived quantity provides the same information about the relation of that quantity to the base quantities as is provided by the SI unit of the derived quantity as a product of powers of the SI base units.

There are some derived quantities *Q* for which the defining equation is such that all of the dimensional exponents in the expression for the dimension of *Q* are zero. This is true, in particular, for any quantity that is defined as the ratio of two quantities of the same kind. Such quantities are described as being dimensionless, *or alternatively* as being of dimension one. The coherent derived unit for such dimensionless quantities is always the number one, 1, *since* it is the ratio of two identical units for two quantities of the same kind [2, p. 106].

Disregarding the difference between VIM3's notion of 'quantity dimension' and SI8's notion of 'dimension', more or less the same things are said in VIM3, but I would like to make one remark. In a note VIM3 rejects SI8's complete equalization of the terms 'dimensionless' and 'dimension one'. Surely, VIM3 accepts both terms, but it regards 'dimensionless' as being obsolete from a strict metrological point of view: "The term 'dimensionless quantity' is commonly used and is kept here for historical reasons [1, p. 6]."

Both SI8 and VIM3 claim that quantities of dimension one and dimensionless quantities have the same metrological unit, one. As far as I can see, the reason for this view is that metrological dimensional products and exponentiations are regarded as being completely analogous to arithmetic products and exponentiations; see the "since" in the second to last line in the SI quotation above. Relative length is then $\mathsf{L}\,\mathsf{L}^{-1} = \mathsf{L}^{0} = 1$, and mass fraction is $\mathsf{M}\,\mathsf{M}^{-1} = \mathsf{M}^{0} = 1$. The weakness of this analogy will be shown in Section 5 below; now some other relevant issues should be discussed.

The metrologist J Valdés, which I will relate to, describes the difference between quantities of dimension one and dimensionless quantities somewhat differently. He connects only the dimension one to the unit one, and connects the term 'dimensionless quantity' to the view that such quantities need no metrological units at all. However, he regards it as only a conventional choice whether to talk of dimension one (and unit one) or of dimensionless quantities (and only pure numbers). He writes (italics added):

We are confronted with two logics:

> Logic A considers the ratio of two quantities as *a new quantity*. If adding two masses m_1 and m_2 we obtain again a mass, then multiplying m_1 by m_2 we obtain a new quantity in kilogram square, and dividing m_1 by m_2 we should also obtain a quantity. Why should division be a special mathematical operation conducting to no quantity at all? The new quantity, though dimensionless, must have a unit. The unit should have a name. [---]
>
> Logic B considers that dividing one quantity by another of the same kind merely expresses the number of times that one quantity fits into the other. [---] If the ratio of both quantities of the same kind is not a new quantity, what is it? It is *just a number*. [---]
>
> Logic A, or Logic B? It is a matter of choice [8, pp. 307-8].

Not all metrologists, however, agree that this is a matter of pragmatic choice. One hard critic of Logic A is W H Emerson [9, 10, 11, 12]. He claims that “All dimensionless quantities now find themselves saddled with, in my view, meaningless and unnecessary ‘units’ [10, p. L27],” and that (italics added) “Of the dimensionless quantities, [---] few people would consider them all to be quantities of the same kind. Differently defined, *dimensionless quantities are not comparable* [11, p. L34].”

From my point of view and terminology, Emerson rests much of his case on the requirement that a unit should be a unit only in relation to a true quantity; in other words, he wants to keep the SI system free from parametric units. If this premise of his is accepted, I find his reasoning conclusive. Let me frame in my own words what I regard as the central objection to the unit one as a non-parametric unit for relative quantities.

Both relative length (original dimension: length/length) and mass fraction (original dimension: mass/mass) should according to the present SI8 system be measured by the same metrological unit, one, and be ascribed the same dimension, dimension one. Now, if one knows only that two rods A and B have the mass fraction 5 mass/mass and that the rods C and D have the fraction 3 mass/mass, then it is impossible to draw any conclusions about the mass fractions A/C and A/D; and the same is of course true if we exchange mass/mass for length/length. Similarly, if A and B has the mass fraction 5 mass/mass and C and D has the relative length 3 length/length, it is impossible to draw any conclusions about a ratio between A and C and A and D. Nonetheless there is a relevant difference between the last example and the first two ones, which brings home Emerson’s point.

Even if not known, *there is* always a mass fraction (and a relative length) also between B and C; and as soon as this fraction becomes known, the mass fractions (and, alternatively, the relative lengths) A/C and A/D can be calculated, and in a significant way compared with that of A/B. However, mass fractions and relative lengths can never in such a sense be compared *with each other*; it is always impossible to compare in a physical-chemical meaningful way A/B mass/mass with, for instance, those of A/B, A/C, and A/D length/length. This means that if both mass fraction and relative length are measured by the unit one, then this unit is a *parametric* unit. Consequently, the dimension one is only a *parametric* dimension.

To my mind, the only way to defend the SI system against Emerson's wish to delete the unit one is to introduce the notion of 'parametric unit', and regard unit one as being a parametric unit. A parametric unit is, to repeat, a unit with a qualitative parameter that needs to be given a specific "value" before it can give rise to meaningful physical-chemical comparisons.

In Section 1, I claimed that mole in effect means mole-of-E_p; in Section 2, similarly, I claimed that joule out of context means joule-of-K_p. What I have now said implies that since the unit one is a parametric unit, it must (when isolated from specific contexts) mean $unit_d/unit_d$, where the subscript d is a parameter for (at bottom) the base property dimensions of the SI system.

(Within parenthesis, I would like to point out that the issue whether dimension one and unit one are true or parametric dimensions and units, must not be conflated with the issue whether or not plane angles can be ascribed the dimension relative length. According to the SI system they can, but according to Emerson [9] they cannot. In his (and my) view, the fact that radian values are defined as the arc *length* of an angle subtended at the center of a circle divided by the radius *length* of the same circle, does not turn the true dimension plane angle into any true dimension length/length, and, therefore, neither into the parametric dimension called 'dimension one'.)

Let me now turn to Logic B. To start with, I will expand the shortened quotation from Valdés, and also insert two explanations (IJ) into it.

> Logic B considers that dividing one quantity by another of the same kind merely expresses the number of times that one quantity fits into the other. This "other" may be the accepted SI unit, in which case the number obtained through division is the result of a measurement, not a new quantity [IJ (i): as when 5 times the

standard meter results in 5 m]. If this "other" is different from the SI unit of the underlying quantity, we may create a second unit for the same quantity [IJ (ii): if rod A is five times rod B and rod B is not measured by an SI unit, we can make rod B a standard length and truly say that rod A has the length 5 rod-B]. If the ratio of both quantities of the same kind is not a new quantity, what is it? It is *just a number* [8, p. 307].

Point (i) makes clear the fact that all the SI8 general quantity values except those for the standard units are relative values; moreover, ratio values, since all SI8 quantities belong to so-called ratio scales (not to so-called 'interval scales' or 'differential property-value scales' [4, chapter 17]). Point (ii) makes clear the fact that in any individual relative quantity such as length-of-A/length-of-B and mass-of-A/mass-of-B, the property instance in the denominator is a *possible* standard unit. This being so, why do some metrologists here want to talk about *pure* numbers? Isn't, by the way, the very term 'dimensionless quantity' a contradiction in terms? Do we not exclusively have either the purely mathematical number line with its numbers or a quantity with its general quantity values (i.e., numbers connected to a metrological unit)? Can there be something in-between? Yes, there can, but I find the term chosen for it, 'dimensionless quantity', very misleading; the adequate name is '*unit-less quantity*'. Let me explain.

Now and then, e.g. in Dybkaer [4] and Emerson [7, 11], quantities (Q) are presented in the symbolic form:

$$Q = \{Q\} \cdot [Q],$$

where $\{Q\}$ represents only a pure numerical value, a number, and $[Q]$ represents only a unit (VIM3: reference). However, in order to account for the fact that one and the same true quantity and true dimension can be ascribed different units, it has to be noted that there is here a third component implicitly present, namely the dimension of Q. The unit $[Q]$ is not just a unit; it is a unit of a certain dimension. Unit transformation formulas such as '1 meter = 1.0936 yards' make sense only if it is assumed that they relate units of the same true dimension; in the case at hand we have '1 meter of *length* = 1.0936 yards of *length*'. There can be no physical-chemically significant transformation formulas for units that belong to different true dimensions. Therefore, when all components of quantities are made explicit (i.e., the dimension is not regarded as hidden in the unit $[Q]$), the symbolic form for an arbitrary quantity looks like this:

$Q = \{Q\} \cdot [Q] \cdot (\dim Q)$.

In statements that report results of measurements this tripartite structure often comes out automatically. Here are some simple examples related to a body called 'b': 'b is 1 m long', 'b has a mass of 2 kg', and 'b has a temperature of 283 K'. In all three cases we find a number $\{Q\}$ (1, 2, 283, respectively), a unit $[Q]$ (m, kg, K), and a dimension (dim Q) (length, mass, temperature).

Let us now, using this symbolism, take a new look at relative length and mass fraction. The numerator and denominator of these quantities (length-of-A/length-of-B and mass-of-A/mass-of-B) belong to the same kind-of-quantity and true dimension, *but no actual general standard unit is needed*, since each of the two magnitudes at hand may be taken as a temporary and local standard for the other one. In such situations, therefore, the unit symbol $[Q]$ can be deleted from the originally tripartite symbolic form, and relative length and mass fraction can symbolically be represented as:

$Q = \{Q_A\}/\{Q_B\} \cdot \dim (\dim Q/\dim Q)$.

If, instead, the short bipartite symbolic formula ($Q = \{Q\} \cdot [Q]$) is used when the unit symbol is taken away, the symbolic form of relative length and mass fraction becomes only:

$Q = \{Q_A\}/\{Q_B\}$.

And here on the surface it really (but wrongly!) looks as if the quantity Q can be identified with only a number ($\{Q_A\}/\{Q_B\}$); and, furthermore (still wrongly!), that it is a quantity that has no dimension at all, and so deserves the name 'dimensionless quantity'. As shown, however, the relative quantities we obtain from the basic tripartite formula have the dimension dim Q/dim Q, and are *not* dimensionless, only unit-less.

The fact that the so-called dimensionless quantities are not dimensionless is in a sense noted in the SI8 brochure (which, remember, claims that dimensionless quantities have the unit one): "In a few cases, however, a special name is given to the unit one, in order to facilitate the identification of the quantity involved [2, p. 120]." Why is there, I ask, a problem with "the *identification* of the quantity involved" if all dimensionless quantities are the same kind of quantity? Why is there a need to distinguish one kind of dimensionless quantity from another? Answer: because the so-called dimensionless quantities are not in fact dimensionless.

Seen from my position, Logic A metrologists rightly think that even ratios of quantities must have a dimension, but then mistakenly think that therefore these quantity ratios have to have a standard unit, too. Logic B metrologists, on the other hand, rightly observe that even ratios that are not purely mathematical can exist as numbers without a unit, but then mistakenly draw the conclusion that such ratios should be considered dimensionless.

Combining my remarks to both these “logics,” the following views emerge: (i) the unit one is in relation to relative quantities superfluous as a true metric unit, since the dimensionless quantities are not in fact dimensionless, only unit-less; (ii) if the unit one and its dimension one is needed anywhere (see Section 6), then the unit one has to be considered a parametric unit and dimension one a parametric dimension. And then I would like to stress a thing about the unit one that the SI8 brochure says (italics added): “The values of all such quantities [quantities of dimension one] are simply expressed as numbers, and *the unit one is not explicitly shown* [2, p. 120].”

To accept both Logic A and Logic B is neither a theoretical solution to the mistake of Logic A nor to that of Logic B. I think both the mistakes displayed might be caused by an insufficient analysis of the relationship between general quantity values and numbers (discussed in Section 4 below) as well as of the relationship between arithmetic multiplications of numbers and metrological multiplications of units and of dimensions (discussed in Section 5).

4. Unit One as an Eighth Base Unit

The SI8 brochure lists exactly seven base quantities and base units [2, pp. 105, 111-116], but in passing there is also an eighth base quantity and base unit acknowledged (italics added):

> There are also some quantities that *cannot be described in terms of the seven base quantities of the SI at all*, but have *the nature of a count*. Examples are number of molecules [---]. Such *counting quantities* are also usually regarded as dimensionless quantities, or quantities of dimension one, with the unit one, 1 [2, pp. 105-6].
>
> All of these *counting quantities* are also described as being dimensionless, or of dimension one, and are taken to have the SI unit one, although the unit of counting quantities cannot be described as a derived unit expressed in terms of the base units of the SI. For such quantities, *the unit one may instead be regarded as a further base unit* [2, p. 120].

The SI system does in this way accept an eighth base quantity, which is either dimensionless or of the dimension called ‘dimension one’; base unit one, symbol 1. Hereby, somewhat remarkably and in a peculiar way, dimension one and unit one are placed both outside of the traditional seven base units system (as in the quotations above) and inside it (as the dimension and unit for relative quantities). But, as I will now show, there is no need to add such a base unit to the SI system, not even as a parametric unit; compare Emerson [7].

Let me start from scratch with some fundamental remarks about the distinction between (purely mathematical) numbers and general quantity values.

If asked ‘please, point at the number 1’, one does not at all know what to point at. In this sense (at least), we do never in the spatiotemporal world meet any numbers. Nonetheless, we meet something related. Instead of numbers we can meet unities of a certain *kind*, i.e., quantity values. Terms such as ‘1 pebble’, ‘2 chairs’, ‘3 flowers’, ‘4 horses’, ‘5 birds’, ‘6 atoms ^{12}C’, ‘7 water molecules’, etc. can be used to refer to entities in the world. And there is no problem in pointing at, for instance, 1 pebble. Bringing in metrology we can say, either that kinds of things such as pebbles, chairs, and molecules *are their own metrological units*, or that in relation to such kinds *there is no distinction to be made between a quantity and its metrological unit*. Linguists call terms of the kind now used ‘count nouns’, and I will call the entities referred to ‘count noun kinds’.

Only entities of the same count noun kind can in a meaningful way be added. Additions such as ‘4 horses + 5 birds’ and ‘6 atoms + 7 molecules’ have no sum that makes sense. However, on a more abstract level a corresponding addition is possible. Since both horses and birds are animals, and ‘animal’ is a count noun, they can *as animals* be added: 4 animals + 5 animals = 9 animals. Similarly, since both atoms and molecules are elementary entities, and ‘elementary entity’ is a count noun, atoms and molecules can *as unspecified elementary entities* be added: 6 elementary entities + 7 elementary entities = 13 elementary entities.

Now, climbing the ladder of abstraction to the very top, we encounter the most abstract count noun kind possible: *entity*. Classified only *as entities*, everything whatsoever can be added. For instance, horses, molecules, nations, dreams, and instances of pain: 4 entities (horses) + 5 entities (molecules) + 21 entities (nations) + 2 entities (dreams) + 13 entities (instances of pain) = 45 entities.

Such abstract entity additions, however, are of no more practical use than additions of purely mathematical numbers, since on this topmost abstract level no kinds of things are differentiated from other kinds of things.[1] Therefore, *there is no need to add to the SI system an eighth base quantity that should take account of entities that have "the nature of a count."* Such a quantity will have no function that is not already taken care of by the numbers. Using the symbolism introduced in Section 3, the quantity expression '7 electrons' fits the unit-less form '$Q = \{7\} \cdot$ (dim electron)'; the expression '5 molecules' fits the form '$Q = \{5\} \cdot$ (dim molecule)'; and '4 horses' fits '$Q = \{4\} \cdot$ (dim horse)'. Every count noun refers to something that from a metrological point of view can be regarded as at the same time both a (true) dimension and a unit.

Terms such as 'water', 'snow', and 'furniture', do not function the way count nouns do; they cannot immediately in a meaningful way be connected to a numeral. Expressions such as '2 water', '3 snow', or '4 furniture' make no sense, and linguists have baptized these terms 'mass nouns'.[2] What mass nouns refer to cannot without further ado be counted. In order to estimate, for instance, amount of water, one has to introduce a unit such as bottle, glass, or molecule. The expressions 'bottle of water', 'glass of water', and 'molecule of water' function the way count nouns do; by means of bottles, glasses, and molecules different amounts of water can be estimated. Out of the term 'furniture' the term 'furniture item' can be created, and as soon as it is decided what counts as a furniture item, then even amount of furniture can be estimated.

Terms for physical-chemical substances such as '^{12}C' and 'H_2O' are out of context ambiguous between being mass nouns and count nouns. But always when they are shorthand for '^{12}C atom' and 'H_2O molecule', respectively, they are count nouns and refer to count noun kinds. This means, among other things, that their amount/quantity/number can be estimated independently of the mole unit; an issue that I will address in Section 6.

All the terms for the base property dimensions in SI8 function the way mass nouns do. Expressions such as '1 length', '1 mass', and '1 temporal duration' are just as meaningless as '1 water' and '1 snow'. But all the terms for the corresponding base units, e.g., 'meter', 'kilogram', and 'second' function the way count nouns do, and refer to count noun kinds. Expressions such as '1 meter', '2 kilogram', and '3 seconds' make perfect sense. The base property units of the SI are their own metrological units. Therefore, in order to be used in practice, no such unit needs to be connected to a further metrological unit such as the unit one.

---

[1] It might be argued that, in fact, the *purely mathematical numbers* 1, 2, 3, etc. are nothing but the *most abstract quantity values* possible, namely 1 entity, 2 entities, 3 entities, etc., but I leave this subtle question to the philosophy of mathematics.

[2] This term 'mass' has of course nothing with the SI base quantity mass to do.

And to claim that entities that have the nature of a count ("counting quantities") need the metrological unit one, is just as unreasonable as to claim that the base property units of the SI ought to be connected to the metrological unit one.

5. Metrological Multiplication Differs Radically from Arithmetic Multiplication

VIM3's definition 1.21 says: "**quantity calculus** = set of mathematical rules and operations applied to **quantities** other than **ordinal quantities** [1, p. 13]." Let me now again from the SI8 brochure quote the following quantity calculus passage about dimensions:

> In general the dimension of any quantity *Q* is written in the form of a dimensional product,
>
> $$\dim Q = \mathrm{L}^{\alpha}\ \mathrm{M}^{\beta}\ \mathrm{T}^{\gamma}\ \mathrm{I}^{\delta}\ \Theta^{\varepsilon}\ \mathrm{N}^{\zeta}\ \mathrm{J}^{\eta}$$
>
> where the exponents α, β, γ, δ, ε, ζ, and η, which are generally small integers which can be positive, negative or zero, are called the dimensional exponents [2, p. 106].

If one sticks to integers (which I think we can do here), it makes good sense to say that arithmetic exponentiation is repeated multiplication ($6^3 = 6 \times 6 \times 6$), and that multiplication is repeated addition (3×5 is equivalent to $3 + 3 + 3 + 3 + 3$ and $5 + 5 + 5$). Arithmetic multiplications of integers have always a clear-cut connection to arithmetic additions, but metrological multiplications have no similar relation to any corresponding metrological addition. Furthermore, they cannot have, since there is no such thing as meaningful additions of physical dimensions or metrological units. For instance, whereas the addition $3 + 3 + 3 + 3 + 3 = 15$ is equal to the multiplication 3×5, the addition 3m + 3m + 3m + 3m + 3m = 15m is not equal to anything that contains a unit multiplication m · m or a dimensional multiplication L · L; compare Emerson [4]. In additions of five quantity instances such as $(\{Q\}\cdot[Q]) + (\{Q\}\cdot[Q]) + (\{Q\}\cdot[Q]) + (\{Q\}\cdot[Q]) + (\{Q\}\cdot[Q])$, the result is always $5 \times \{Q\} \cdot [Q]$; the five instances of the metrological unit $[Q]$ are *not* added.

This clear difference between arithmetic and metrological multiplication means that what is true of arithmetic multiplications (e.g., $a\, a^{-1} = a^0 = 1$, $a \neq 0$) need not necessarily be true also of metrological multiplications. If no special reasons are supplied, metrological multiplications such as m m^{-1} and L L^{-1} cannot be said necessarily to be equal to $m^0 = 1$ and $L^0 = 1$, respectively. The SI8 brochure, however, seems to be of another opinion (italics added):

> Certain quantities are defined as the ratios of two quantities of the same kind, and are *thus* dimensionless, or have a dimension that may be expressed by the number one. The coherent SI unit of all such dimensionless quantities, or quantities of dimension one, is the number one, since the unit *must be the ratio* of two identical SI units. The values of all such quantities are simply expressed as numbers, and *the unit one is not explicitly shown.* [2, p. 120]

Since so far I have nowhere seen any special reasons in support of the view "*must be the ratio*," I think that metrologists have better regard the dimensions of relative length and mass fraction as being the un-reducible dimensions L L^{-1} and M M^{-1}, respectively. Put generally:

- the dimension (dim *Q*/dim *Q*) equals neither the dimension one nor the pure number 1, it is un-reducibly dim *Q*/dim *Q*.

This view means also that metrological dimension multiplications such as $(\dim Q)^3 (\dim Q)^{-2}$ are not equal to (dim *Q*); they are nothing but $(\dim Q)^3 (\dim Q)^{-2}$. For instance, the dimension of rainfall is volume per area, not length; its unit is m^3 m^{-2} not m, even though many rain-gauges report rainfall quantities in values such as 'millimeter rain'. More words about this issue (and the next one) can be found in Emerson [11, 12].

What then about multiplications of dimensions and units that do not give rise to relative quantities? What about, for instance, L^2 with unit m^2, L T^{-1} with unit m s^{-1}, and M L^{-3} with unit kg m^{-3}? How can they be dimensions and units for area, speed, and density, respectively? In my opinion, *out of context*, they cannot. When area is ascribed the dimension L^2, the context makes it clear that the two L has to be orthogonal to each other, something which is not and cannot be said in the SI, since it restricts itself to scalar quantities. When speed is ascribed the dimension L T^{-1}, the context has it that L and T are related to one and the same movement. When density is ascribed the dimension M L^{-3}, the context makes it clear that M and L^{-3} are properties of one and the same material thing. In abstraction, these metrological multiplications have no specific physical-chemical significance.

PART III: The Parametric Unit Mole

6. Rename 'Amount of Substance' to 'Elementary Entities' and Exchange the Mole for the Unit One

I will now connect back to Section 1. To accept that amount of substance is a parametric quantity and its unit mole a parametric unit, is to open up for a new look at this quantity and its unit in other respects, too.

In the 1977 paper "Amount of substance and the mole" [13], the chemist and metrologist M L McGlashan found it necessary to write a paper in which he tried to correct wrong interpretations of the newly in the SI introduced mole. I quote:

> Although widely used by chemists, the physical quantity called amount of substance and its SI unit called the mole are not necessary in science. It would be perfectly feasible to deal always with molecular quantities and at least in physics that is often done. [---] Nevertheless, for historical reason it is customary in chemistry (and in physics too) to use the redundant physical quantity amount of substance and its SI unit the mole. So long as they are used they should be used correctly. The rest of this article will be about their correct use [13, p. 276].

However, outside of chemistry, the mole has continued to suffer from misinterpretations that no other base unit has suffered from. For instance, at least as late as February 2008 (but later corrected), *Wikipedia* falsely said that "A mole is much like 'a dozen' in that both are absolute numbers (having no units)", and another internet dictionary *Whatis.com* still says (February 2010) that "The mole is the only fundamental SI unit that is dimensionless." That is, in spite of the fact that all the editions of the SI unambiguously proclaim that the mole is a unit and not a pure number, and that it has a dimension, namely amount of substance, well-meaning people have made claims to the contrary. I hope to be able to show that some fault is on both sides; neither the SI metrologists nor the quoted internet metrologists use the notions of 'parametric unit' and 'parametric quantity', but this is needed if one should not get lost.

The first of the two paragraphs in which the SI brochure introduces the mole can also, since the number of atoms in 0.012 kilogram of carbon 12 is known and equal to the Avogadro number (a purely mathematical entity that I will symbolize A_N), be stated thus:

> 1*. 1 mole is the amount of substance of a system which contains A_N elementary entities; its symbol is “mol”.

And many scientists present the mole in this way. Here is a quotation from one well established introduction to physical chemistry: “A ***mole*** of some substance is defined as an amount of that substance which contains Avogadro’s number of elementary units” [14, p. 9].

Let me again stress that it is here spoken of the Avogadro *number* A_N (= $6.022\ 14 \times 10^{23}$); not the Avogadro *constant* N_A, which is a number connected to the parametric unit mole ($N_A = 6.022\ 14 \times 10^{23}$ mole^{-1}). The relation between the Avogadro constant and the Avogadro number is: $N_A = A_N$ mole^{-1}.

The importance of the distinction between the Avogadro constant and the number is easily shown. In definition 1*, the phrase ‘contains A_N elementary entities’ cannot be exchanged for ‘contains N_A (= A_N mole^{-1}) elementary entities’, since this would make the definition circular; the mole would in such a case appear not only as that what is to be defined, but in the defining expression, too.

Since 1 mole is the amount of substance of a system which contains A_N number of elementary entities, the mole can be as directly connected to the Avogadro number as to the Avogadro constant. And it has been argued that the former connection should be made the primary one:

> It is proposed that the Avogadro constant be converted to a number, the ‘Avogadro number’, and that the mole be linked to this number. The unit of the amount-of-substance would be this particular number of specified, identical entities. This would not only bring greater clarity and simplicity to the SI, but would also lead to a better understanding of the mole by the physics and chemistry communities, as well as by the general public. [15, p. 11]

But if this proposal is combined with the insight that the mole is a parametric unit, then also another proposal becomes reasonable: *rename the parametric quantity and dimension called ‘amount of substance’ to ‘elementary entities’*. Let me explain the route.

An application of the term ‘amount of substance’ requires (see Section 1) an expression of the form ‘amount (of substance) of elementary entities of kind p’. Here, first, the parenthesis can be deleted; this is explicitly said in the SI8 brochure [2, p. 115]. Second, if an actual application value is added, an expression of the form ‘*n* mole amount of elementary entities of kind p’ appears. But here, third, the phrase ‘amount of’ is redundant; the expression ‘*n* mole elementary entities of kind p’ conveys the same information as the longer expression. Therefore, fourth, the parametric quantity and dimension amount of substance could just as well be called ‘elementary entities’. The plural form might be taken as indicating that ‘elementary entities’ names a parametric quantity and dimension, i.e., a quantity and dimension that necessarily refers to other but true metric quantities and dimensions. If this renaming proposal is accepted, definition 1* can be replaced by this one:

> 1**. 1 mole is the number of elementary entities of a system which contains A_N such entities; its symbol is “mol”.

When the fact that the mole is a parametric unit is made explicit, i.e., E_p is introduced, the following equalities appear (below, n is only a variable for mole-of-E_p and N only for elementary entities of kind E_p):

- n mole-of-E_p = N entities-of-E_p / A_N
- n mole-of-E_p = $(n \times A_N)$ E_p
- 1 mole-of-E_p = A_N E_p

It may be argued, however, that these equalities do not meet the requirement that both the sides of such quantity equalities must have the same dimension. Let us look at the last equality (1 mole-of-E_p = A_N E_p). On the left hand side there is the unit mole and its dimension, but on the right hand side there seems to be no unit at all. Now, if the mole on the left hand side is not a parametric unit but a true unit on a par with the other base units, metrology says that we cannot possibly have the Avogadro number A_N on the right side; the number A_N has to be exchanged for the Avogadro constant N_A. So far, I have no objections. But there is more to be said.

Let now the mole be what it really is, namely a parametric unit. Then 1 mole-of-E_p = A_N E_p passes the dimension test if there is a corresponding parametric unit on the right side. And we can, at least without immediately doing violence to the SI8 brochure, try to insert one, namely the parametric unit one discussed in Sections 3 and 4. That is, we can try to *reverse* the SI8 statement that “values of all such quantities [quantities of dimension one] are simply expressed as numbers, and *the unit one is not explicitly shown* [2, p. 120].” We can *insert* unit one, whereby we obtain:

- 1 mole-of-E_p = A_N one-of-E_p

If the unit one is, as here, allowed to have elementary entities as its dimension – and nothing tells against that – then the equality does meet the dimension test; on both the left and the right hand side of the equality we find the dimension elementary entities. This fact shows that the mentioned *Wikipedia* mistake (the mole is only a mathematical number) and the *Whatis.com* mistake (the mole has no dimension) were not completely ungrounded. If one looks at the equality 1 mole-of-E_p = A_N E_p, and takes the right hand side both literally (i.e., as having no unit one hidden) and as pre-given, then the mole must be just a number, and so be lacking a dimension, too. Really, the sentence ‘1 mole-of-E_p = A_N E_p’ is in appearance structurally similar to the sentence ‘1 dozen-of-E_p = 12 E_p’, which had a counterpart in the Wikipedia article referred to. If the SI system had contained a distinction between true and parametric units, these internet metrology mistakes could have been avoided.

If one starts to think in terms of parametric quantities and parametric units, it seems to me as if a proposal such as the one below comes naturally; and has, outside of some areas within physical chemistry, great pedagogical advantages:

(i) rename in the SI the parametric quantity now called ‘amount of substance’ to ‘elementary entities’ (the symbols *n* and N need not be changed);

(ii) exchange in the SI the parametric unit mole-of-E_p for the parametric unit one-of-E_p; use then when needed the unit transformation formula 1 one-of-E_p = $1/A_N$ mole-of-E_p;

(iii) use the existing SI8 recommendation to delete the unit one, and write always ‘1 one-of-E_p’ as ‘1 E_p’ (i.e., talk only of numbers and specified kinds of elementary entities is enough).

In my opinion, if accepted, this proposal ought to have the consequence that mole-of-E_p suffers the same fate in relation to E_p as, for instance, the units minute, hour, and day have done in relation to second, and litre in relation to meter^3. That is, the mole becomes a non-SI unit accepted for use with the SI. So understood, *the proposal does not in any way imply that chemists have to stop using the unit mole*.

Let me, however, by means of one single example show what a complete drop of the mole might imply. The gas constant R in the ideal gas law is today ascribed the unit joule per kelvin per mole (J K^{-1} (mol-of-$E_p)^{-1}$); according to the proposal, R should instead be ascribed the non-coherent unit joule per kelvin per Avogadro number of elementary entities (J K^{-1} (A_N $E_p)^{-1}$). This, in turn, means that the mole formulation of the gas law would be deleted in favor of the formulation containing the Boltzmann constant.

What has just been said also sheds light on the unexplained notion of 'universal constant' that figures on one single page in SI8 and never in VIM3. The equality $n(X) = N(X)/N_A$ in the quotation below means n mole of X = N entities of X divided by the Avogadro constant, i.e., we find here the Avogadro constant with the parametric dimension mole^{-1}. Here is the paragraph:

> The definition of the mole also determines the value of the universal constant that relates the number of entities to amount of substance for any sample. This constant is called the Avogadro constant, symbol N_A or L. If $N(X)$ denotes the number of entities X in a specified sample, and if $n(X)$ denotes the amount of substance of entities X in the same sample, the relation is
>
> $$n(X) = N(X)/N_A.$$
>
> Note that since $N(X)$ is dimensionless, and $n(X)$ has the SI unit mole, the Avogadro constant has the coherent SI unit reciprocal mole [2, p. 115].

The equality $n(X) = N(X)/N_A$ is no natural law; it is only a result of the stipulation that n is a variable for moles and N a variable for number of entities. The Avogadro constant N_A is called a *universal constant*, but this does not mean that in science it plays the role of a *fundamental constant* such as the Planck constant and the constant c_0, the speed of light in vacuum. The fact that the Avogadro constant figures in the transformation formula between the gas constant (R) and the Boltzmann constant (k), $k = R/N_A$, does not turn it into a fundamental constant. This transformation formula merely reflects the fact that $pV = nRT$ and $pV = NkT$ are two different but equivalent formulations of one and the same natural law, the ideal gas law. The mole formulation of this law is by no means a theoretical necessity that has to be retained whenever the ideal gas law is on the agenda.

The view that the mole is only a parametric unit is, I think, worth thinking through; it can affect traditional metrology.

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