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**ESSAY REVIEW**

# Applied mereology

## C. Calosi and P. Graziani (eds.): Mereology and the Sciences. Parts and Wholes in the Contemporary Scientific Context
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**Ingvar Johansson**

Professor emeritus in theoretical philosophy at Umeå University, Sweden
e-mail: ingvar.johansson@philos.umu.se

One purely formal discipline, mathematics, permeates modern natural, social, and medical sciences as well as technology and engineering. Another purely formal discipline, formal mereology, has the last decades rapidly grown within philosophy. The obvious question now is: *of what use can formal mereology be in the sciences and in engineering?* In philosophy this question has seldom so far been posed, but now there has appeared an anthology in which the editors C. Calosi and P. Graziani rightly state: “It should be clear that the present volume fills indeed the gap between philosophical reflections on parts and wholes and the contemporary scientific context. Or better, it begins to fill this gap” (*Mereology and the Sciences*, p. xvii). The book is truly to be welcomed.

The editors have divided the contributions into four parts: I “Physics” (4 papers), II “Mathematics” (3 papers), III “Natural Sciences” (2 papers), and IV “Computer Sciences and Engineering” (3 papers). Each part is preceded by a brief two pages introduction by the editors. There is also a selected bibliography and an appendix, “Formal Theories of Parthood”, written by one of the main figures behind the expansion of mereology within contemporary philosophy, Achille C. Varzi.

(For a detailed exposition of the contents, see: http://link.springer.com/book/10.1007/978-3-319-05356-1.)

The main thing I will argue for in this essay review is, that if a clear distinction is made between *formal* (or pure ) mereology and *applied* mereology, then what is going on in the volume can better be seen. This distinction is needed not only in relation to part-whole conceptions in the sciences, but also in relation to what the editors call “philosophical reflections on parts and wholes”. When in analytic metaphysics formal mereology is put in the service of metaphysics, it should just as in the sciences be regarded as applied mereology. The first two papers in the physics section are about the notions of endurant and perdurant entities, which are much discussed in physics oriented analytic metaphysics, but not in physics itself. In order to pave the way for an introduction of my distinction between formal (pure) and applied mereology, I will first make a brief presentation of the distinction between pure and applied arithmetic.

Whatever the true ontological nature of the *pure* natural numbers is, it is impossible to point to them in the spatiotemporal world we perceive. However, as soon as a count noun is available (e.g. ‘chair’ and ‘molecule’) then *applied* arithmetic immediately becomes possible. To each referent of a count noun the numeral ‘1’ is applicable, and when such referents are perceivable, one can point to one or a certain number of entities of that kind. With mass nouns (e.g. 'furniture' and 'water') things are not that easy, but there are two ways in which arithmetic can be made applicable even in relation to such nouns. *Either* the mass noun is defined by means of count nouns (e.g., 'furniture' $=_{df}$ 'chair' or 'table' or 'lamp' etc., and 'water' $=_{df}$ 'collections of water molecules'), *or* a conventional unit is introduced in order to estimate the amount of the referents of mass nouns. In the case of water, one may in everyday life use a certain kind of glass or a certain kind of bottle as a unit. Then one may speak of the amount/quantity to be estimated as '*x* glasses of water' or '*x* bottles of water'. Similar introductions of a conventional unit are also needed in relation to the kind of continuous quantities that abound in mathematical physics: length, mass, force, electric current, energy, etc. I am using 'applied' in such a wide sense that both theoretical and experimental physics are cases of applied mathematics.

Since every parthood relation in the world is a certain kind of parthood, it is as impossible to point to any *pure* parthood relations in the world, as it is to point to any pure numbers. Therefore, even mereology needs like arithmetic a distinction between pure (formal) and applied theorizing. Just as numerals must be connected to count nouns (or something that functions as such) in order to be able to refer to something in the world, so the parthood predicate of formal mereology must be connected to adjectives such as 'spatial', 'temporal', and 'functional' before they can refer to parthood relations in the world. The really important point, however, comes next: *the move from formal to applied mereology is not always unproblematic*.

The contrary position, that applying mereology is always a trivial task, may have several causes, but I will only mention what I think is the central one. Most mereologists seem to have a view that is expressed thus: "we shall be working under the following simplifying assumptions: […] *Monism*: There is a single relation of parthood that applies to every entity independently of its ontological category" (Varzi, p. 359). I find this common wording of the monistic assumption ambiguous.

It may mean that parthood relations can exist in many different ontological realms such as the material, the mental, the abstract, and the fictional, and also in different ontological categories such as objects, properties, and relations; but then the word 'every' had better been deleted. It would be better to use only the expression 'can apply to entities independently of ontological category'. The presence of 'every' makes it possible to read the assumption as instead stating that every entity whatsoever is the relata of some parthood relation. If this is the case, then there are of course no general application problems, but it is only the first interpretation that is consistent with the whole of Varzi's presentation of formal mereology.

The ambiguity between 'being applicable in different ontological domains' and 'being applicable everywhere', which I have highlighted, normally inflicts the notion 'formal ontology', too. And many mereologists look upon themselves as doing formal ontology. The editors write: "Formal ontology attempts at lying down the bare formal structure *of all there is* [italics added], whatever there is. On this account, regardless of what entities should be admitted in our domains of quantification, all there is must exhibit some general structure and obey some general laws, and the task of [formal] ontology would be to describe such structures and laws" (p. xv). Do they want to say that formal ontology, despite being a non-empirical discipline, is applicable everywhere in the world, or only that it is not by its nature confined to certain specific ontological domains? I don't know.

There exists a number of consistent formal axiom systems for the parthood relation. In the book these mereolog*ies* are given a very elegant presentation by Varzi. Using the predicate 'proper parthood' (*PP*) as the primitive term, the following holds true. *Ground* mereology consists of three axioms (*PP* is irreflexive, asymmetric, and transitive). By adding to it the axiom of weak supplementation (informally: a whole cannot have only one proper part), *minimal* mereology is obtained; and by adding further specific axioms *extensional* mereology and *general extensional* mereology are obtained. The last mereology is also called *classical* mereology.

Varzi maintains (see quotation above) that *all* formal axiom systems for parthood refer to one and the same two-place parthood relation. This implies that if classical mereology is by further axioms extended into new kinds of mereologies, then even these new systems must refer to the same old parthood relation. Now an interesting observation can be made. Without any internal inconsistencies arising, classical mereology can be extended either by an axiom stating that there are partless parts (mereological atoms), or by the opposite axiom that there are no mereological atoms (all parts have parts), (Varzi, p. 363). Even though both of these extensions of classical mereology are consistent systems, both cannot on pain of contradiction be applied to the whole or to some specific part of spacetime. No part of spacetime can possibly be both atomistic and "gunky". Therefore, the parthood relation talked about in these systems cannot in itself be about a this-worldly parthood relation; it needs mediation. To note this is to note something that has become obvious in the history of geometry.

From Euclid to Kant it was normally taken for granted that Euclidean geometry directly specifies the structure of space. The nineteenth century mathematical constructions of the Non-Euclidean geometries meant a death blow to this view. The parallel axiom of Euclidean geometry could be exchanged for other axioms without any internal inconsistencies arising. (The other common axioms should now be thought of as corresponding to classical mereology, which is silent on the topic of mereological atoms.) Nonetheless, mathematicians have continued to regard the notion of straight line (important in the parallel axiom and its substitutes, but mentioned also in some others) to be the same in all geometric axiom systems. Therefore, the question whether the spatiotemporal world has a Euclidean or some Non-Euclidean structure cannot be answered by pure mathematical geometry. In order to answer it, one needs a logically prior decision as to what is to count as a straight line in the spatiotemporal world. As there are no pure arithmetic numbers in the world, there are no pure geometric lines (i.e., in themselves infinitely thin spatial lines) either. If light rays are regarded as moving in straight lines, then spacetime as a whole has a Non-Euclidean structure, whereas every infinitesimal part has a Euclidean structure.

The point I am trying to bring home, that the parthood relation of classical formal mereology is *not* directly about spatiotemporal entities, comes out implicitly also in another way in Varzi's exposition. Classical mereology cannot without contradiction be extended by an axiom saying that every whole is part of a greater whole (Varzi, p. 369). This implies, that to regard classical mereology as directly being true of our spatiotemporal world is to claim, that we a priori know that this world has no largest whole (i.e., that it is not "junky") or that it is not a part of something else, e.g. God (as the panentheists claim). I am neither a "junkyist" nor a panentheist, but I don't think these my views are provable by formal mereology.

I will now show how the lack of the introduced distinction between formal and applied mereology affects various things said and made in the book. I will go through the papers part by part.

The first author, C. Gilmore, says: "one can get this far while confining oneself to the *perdurantist*'s attractively simple fundamental ideology – a primitive two-place predicate for parthood *simpliciter*" (p. 33). Perdurantists are discussing spacetime parthood, not parthood *simpliciter*; the former belongs to applied mereology, the latter to formal mereology. The editors say about Gilmore's contribution that it "entails the *rejection* [italics added] of two principles of classical mereology, namely extensionality and unrestricted composition" (p. 2). I think they should have said: 'entails the *non-applicability* of two principles of classical mereology'. In the same unfortunate way of writing they also say that Y. Balashov's contribution "entails the rejection of the unrestricted composition principle" (p. 2).

C. Calosi & G. Tarozzi make a distinction between mereology and models of mereology, which brings them very close to my distinction between formal and applied mereology. They argue that quantum systems are, despite claims to the contrary, models of classical mereology, but they never make explicit the fact that the parthood predicate of the model must differ from the parthood predicate of formal mereology.

V. Fano & P. Graziani argue that a "gunky" mereology is inconsistent with classical mechanics. Another way to put it would be to say, that a "gunky" formal mereology cannot be applied where the theory of classical mechanics is applicable.

When the relation between formal mereology and mathematics is truly discussed, as by C. Coppola & G. Gerla and R. Gruszczynski & A. Pietruszczak, there is of course no need to bring in the distinction between formal and applied mereology. It is then a matter of relating different kinds of purely formal systems to each other. However, one of the papers in the mathematics part, "Natural Mereology and Classical Mereology", breaks out of the formal confinement of the section heading. The author, P. Hovda, opposes to each other two philosophical orientations towards mereology, a naturalistic and a formalistic orientation, respectively. The former contains the view "that reality includes a great deal of *natural mereological structure*, which must be discovered (at least partly) by empirical means, and for which there is no a priori reason to think that it will fit any neat formal pattern" (p. 141). Formalistic mereology claims that there is "an a priori science of mereology whose truths reveal a great deal about the overall *pattern* of part-whole connections in the universe. Crudely, we may take this view to be that Classical Mereology (or some similar formal theory) give the one true theory of the part-whole relation" (ibid.).

Whereas I have introduced a distinction between two *non-competing* kinds of mereology, applied and formal, Hovda makes a distinction between two *competing* philosophical ways of looking at mereology, a naturalistic (and a posteriori) and a formalistic (and a priori). Interestingly enough, to my mind, he does nonetheless not want to choose between these two competing views. He tries to solve the opposition by rejecting both and creating a new mereological system; one that takes features from both the naturalistic and the formalistic orientation into account. However, as he himself admits, this attempt at a synthesis runs at least into one specific trouble: "in a very clear sense, the *only* possible failing [of his formal device] is that the resulting [parthood] relation might not be [as in ground mereology] anti-symmetric" (p. 146). I think this failing is not accidental. Let me explain.

Normally, mereologists do not bother about whether 'part' (*P*) or 'proper part' (*PP*) should be taken as the primitive predicate. As far as I can see, the reason is that in *formal* mereology it doesn't really matter if one makes *P* or *PP* the primitive term by means of which the other is defined. However, in *applied* mereology it does matter. In applied mereology one ought always to regard *PP* as primitive. Why? It is an empirical question what kind of proper parts a whole has, but that a whole is identical to itself is a logical truth. Therefore, it is only by making *PP* and 'identity' primitive terms, and *P* a defined term ($Pxy =_{\mathrm{df}} PPxy \vee x{=}y$), that the epistemological difference that in applied mereology exists between proper parthood and identity is not obfuscated. If someone claims that *Pab*, one should know whether it is an empirical or a logical claim. In formal mereology, on the other hand, such epistemological issues are of no concern; they are abstracted away. Now, since *P* is anti-symmetric but *PP* asymmetric, it is to me no wonder that Hovda gets into trouble with anti-symmetry. He uses *P* as his primitive term, but in "natural mereology" *PP* is what counts.

It is often hard to state exactly where the boundaries of particular spatiotemporal parts (e.g. the heart as part of the body) are situated, and this is well known. However, but too seldom noted, even the very notion of part is in some contexts in need of being given a clear boundary. In the volume's biology paper, by L. Jansen & S. Schulz, this problem is highlighted. In biology, the parthood notion is *not* extensionally equivalent with either the notion of containment or that of inclusion, even if when applied to relations between spacetimes regions, the notions differ only in intension.

The chemistry paper, by J-P. Llored & R. Harré, revitalizes an issue that was brought into mereology in the fifties by N. Rescher. He claimed that there are parthood relations that are *not* transitive, and so do L&H, but no other contributor to the volume. If true, it means that not even ground mereology is applicable to all kinds of parthood relations. Here is one example going back to Rescher: a platoon is part of a company, and a company is part of a battalion, but yet a platoon is not part of a battalion. L&H set all classic examples aside, and bring in examples from contemporary chemistry to prove their point. But before saying more about their position, I will take the opportunity briefly to present my analysis of the classic examples (for papers, see the book's selected bibliography).

Consider the predicate '*large* spatial part'. It is not (unlike 'spatial part') transitive. Does then the existence of large spatial parts show that there are parthood cases where the transitivity axiom of ground mereology becomes falsified? No, it does not, but it shows that there are parthood cases where the axiom is inapplicable. Falsifications require meaningful applications. Let me explain.

The parthood relation of classical formal mereology is a two-place relation. It is a category mistake to apply it to spatiotemporal $n$-place relations where $n > 2$. Being a large spatial part, however, is at least a three-place relation. In order for a part of a whole to be reckoned large, it has to be compared to some third entity different both from itself and from the whole. Therefore, no cases of the (at least) three-place relation large spatial parts can falsify the transitivity axiom for two-place parthood. But this fact creates another question. Is it perhaps a mistake to think, as I have now done, that there can be three-place parthood relations? Perhaps all true parthood relations must be two-place relations? My impression is that most mereologists treat the parthood notion as if it can only encompass two-place parthood, but I think this is wrong.

I have already used one crucial event in the history of mathematics, the discovery of Non-Euclidean geometries, in order to shed light on formal mereology. I will now use another, the discovery of the irrational numbers.

The first Pythagoreans took it for granted that all spatial extensions must be comparable by means of the natural numbers. Put in modern terms, all comparisons between two spatial extensions can be expressed by means of a rational number. Then, famously, it was discovered that this is not always the case. The ratio between the hypotenuse and a side in a right-angled equilateral triangle cannot be expressed by a rational number. At first, many Pythagoreans refused to believe it, but eventually all had to give in and accept that there exists a hitherto unknown kind of numbers, which became labeled 'irrational numbers'.

In my opinion, mereologists who claim that, necessarily, all parthood relations are two-place transitive relations behave a bit like the early Pythagoreans. I think intransitive parthood relations should be accepted, but it should be noted that they are at least three-place relations. We may call the former *simple* and the latter *complex* parthood relations. Simple parthood is then a two-place relation that necessarily is transitive, but complex parthood is at least a three-place relation, and it need not be transitive.

Many of the classic examples of intransitvity of parthood can be subsumed under the notion 'functional parthood' (for instance, a platoon is a functional part of a company). In order for something to be a functional part, however, it is not enough to have a relation to a functional whole; the part must also have an interactive relation to some other functional part. Being a functional part is a case of complex parthood. The predicate 'functional part' is just like 'large spatial part' (at least) a three-place relation, a part-*part*-whole relation. This explains to me the intransitivity of functional parthood. Whether any interesting formal mereological axiom systems can be made to conform to a three-place parthood predicate is a secondary issue.

The importantly new thing with L&H's paper is that they make their non-transitivity case not by focusing on part-*part*-whole relations, but on what I would like to call part-*part*-whole-environment relations. That is, they focus on relations that are (at least) four-place relations. I quote:

> In quantum chemistry, the mereology used by chemists entangles the whole, its parts, and, sometimes, its environment. It is not a classical transitive mereology in so far as the whole interacts with its environments. [---] Quantum chemical practices need the whole, the parts, and the other entities at the same time, [---] The whole needs other wholes to be defined, not just intrinsic parts. In this respect, philosophers need a relational mereology not a classic one which cuts off the whole from its environments. (p. 195)

The papers in the last part of the book, "Computer Sciences and Engineering" (by L. Polkowski, A. Galton, and D. Bjørner, respectively) are of course in fact concerned with what I have called applied mereology, but this is, at least in the first two papers, not at all made clear. The authors are without any qualifications using the parthood predicate of formal mereology. The third author explicitly confines himself to spatial parthood (p. 323), and presents his general perspective on mereology as follows:

> Mereology, to us, is the study and knowledge about how physical and conceptual parts relate and what it means for a part to be related to another part: *being disjoint*, *being adjacent*, *being neighbours*, *being contained properly within*, *being properly overlapped with*, etcetera. By physical parts we mean such spatial individuals which can be pointed to. (p. 326)

I think mereologists of all kinds have reasons to reflect on the distinction between formal and applied mereology.